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Physics of Electrical Discharges

Reference

Mechanism of Electrical Discharges
Chapter 3
The Lightning Flash (Edited by V. Cooray)
The Institute of Electrical Engineers, 2003
ISBN 0 85296 780 2

Introduction to Lightning (V. Cooray)
Springer, 2015
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Physics of Electrical Discharges

Unlike other branches of physics, Physics of Electrical Discharges is not an exact science. At least NOT YET.

The problem is the statistical nature of the discharge mechanism.

We cannot predict exactly, or give the exact conditions under which the electrical breakdown in air will take place.

Applications of physics of electrical discharges

- Initiation, propagation and effects of normal as well as upper atmospheric lightning flashes
- Interaction of lightning flashes and other electrical electrical discharges with power transmission and telecommunication systems.
- Interaction of ESD with integrated circuits and ESD initiated explosions.
- Industrial applications of electrical discharges (electrostatic precipitation, water purification, spray painting etc.)

Let us start with some basic definitions

Collisions and Mean free path

An electron moving in a medium consists of other atoms can make either elastic (kinetic energy is conserved) or inelastic collisions (part or all of the kinetic energy becomes potential energy).

An inelastic collision between an electron and an atom could lead either to the attachment of the electron to the atom, to the excitation (electronic, vibrational or rotational) of the atom or to the ionisation of the atom.

The distance an electron travels between elastic collisions is called the free path for elastic collision. Average value of this is the mean free path for elastic collision.

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Mean free path and Cross section

The mean free path for a given type of collision can be described by the equation

$$\lambda = \frac{1}{n\sigma}$$

n : density of gas

σ : microscopic collision cross section (dimension- area)

$n\sigma$: macroscopic collision cross section of the process under consideration.

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Distribution of free paths

A stream of electrons enter into a medium with gas molecules at *x* = *0*. Each time an electron collides with an atom it is removed from the stream. Let λ be the mean free path of the electrons.

At the location *x* the number of electrons that survived without making a collision is *n(x).* The number of electrons lost in moving from *x* to *x*+*dx* is

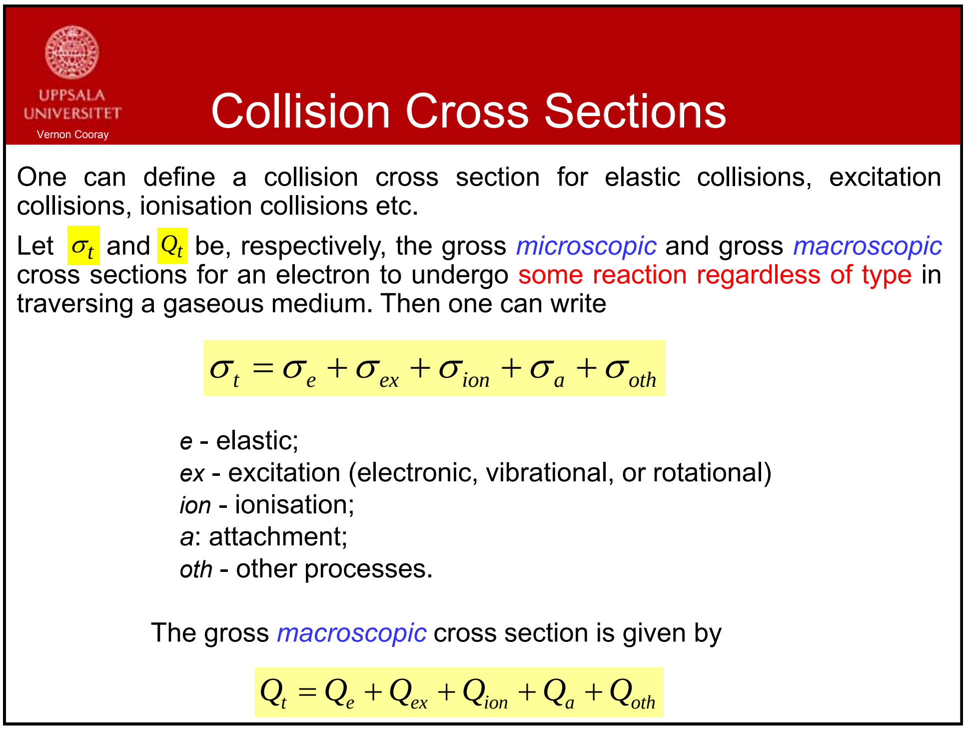
$$dn = -\frac{dx}{\lambda} n(x) \qquad x = 0;\ n = n_0$$

The solution of this equation is

$$n(x) = n_0 e^{-x/\lambda}$$

The probability of an electron having a free path larger than *x* is

$$\frac{n(x)}{n_0} = e^{-x/\lambda}$$



Collision Cross Sections

One can define a collision cross section for elastic collisions, excitation collisions, ionisation collisions etc.

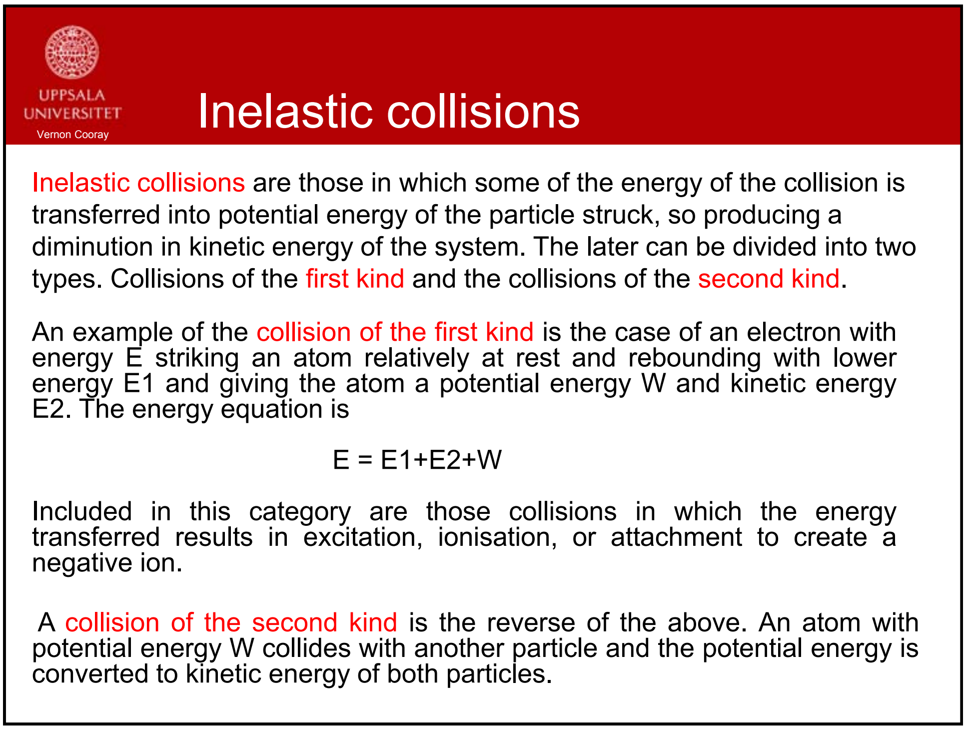
Let σ_t and Q_t be, respectively, the gross *microscopic* and gross *macroscopic* cross sections for an electron to undergo some reaction regardless of type in traversing a gaseous medium. Then one can write

$$\sigma_t = \sigma_e + \sigma_{ex} + \sigma_{ion} + \sigma_a + \sigma_{oth}$$

e - elastic;
ex - excitation (electronic, vibrational, or rotational)
ion - ionisation;
a: attachment;
oth - other processes.

The gross *macroscopic* cross section is given by

$$Q_t = Q_e + Q_{ex} + Q_{ion} + Q_a + Q_{oth}$$



Inelastic collisions

Inelastic collisions are those in which some of the energy of the collision is transferred into potential energy of the particle struck, so producing a diminution in kinetic energy of the system. The later can be divided into two types. Collisions of the first kind and the collisions of the second kind.

An example of the collision of the first kind is the case of an electron with energy E striking an atom relatively at rest and rebounding with lower energy E1 and giving the atom a potential energy W and kinetic energy E2. The energy equation is

$$E = E1+E2+W$$

Included in this category are those collisions in which the energy transferred results in excitation, ionisation, or attachment to create a negative ion.

A collision of the second kind is the reverse of the above. An atom with potential energy W collides with another particle and the potential energy is converted to kinetic energy of both particles.

The maximum energy that can be transferred during a collision

Consider two particles m1 and m2. The particle m2 is initially at rest and the particle m1 is moving with a speed v1. Let U be the change in the internal energy of the particles during collision.

Following conservation laws

$$\frac{m_1 v_1^2}{2} = \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} + U \qquad m_1 v_1 = m_1 V_1 + m_2 V_2$$

One can show that the maximum possible value of U is given by

$$U_m = \left(\frac{m_2}{m_1 + m_2} \right) \frac{m_1 v_1^2}{2}$$

One can see for a given amount of energy, an electron can transfer almost all its energy to the potential energy of the other particle whereas an ion can transfer only about half of its energy.

Drift velocity: V_d

In a vacuum the charged particles will continue to accelerate under the influence of the electric field but in a medium full of gas atoms the charged particles will make collisions with the atoms resulting in a loss of energy gained from the electric field.

As a result the charged particles will attain a certain constant speed within a certain time called the *relaxation time*.

This constant speed is known as the *drift velocity*. The drift velocity depends on the applied electric field, the charge and the mass of the particle among other parameters.

$$V_d = \mu E$$

μ is called the mobility of the particle

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What is electrical breakdown?

The electrical breakdown of air takes place when the air changes from an insulator to a conductor.

This process is mediated by an increase in the electron concentration in air. The processes that lead to the increase in electron concentration in air are called ionisation processes.

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Ionization processes

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Ionisation due to electron impact

In an electric field an electron continues to gain energy but it can transfer only a quantum of its energy to atoms in the medium during an inelastic collision.

Thus when the electron energy increases more than the excitation energy of the colliding atom a quantum of energy is transferred during collisions leaving the atom in an excited state.

If the electron energy is larger than the ionisation energy of the atom a collision may result in the ionisation of the atom.

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Quantification of ionization due to electron impacts

The process of ionisation due to electrons can be quantified either in terms of an ionisation cross section, coefficient of ionisation or the probability of ionisation.

The coefficient of ionisation (also known as the Townsend's primary ionisation coefficient) is defined as the number of ionisation collisions made by an electron in moving a unit distance along the direction of the electric field. Usually this is denoted by the symbol α.

The probability of ionisation is defined as the ratio of the number of ionisation collisions to the total number of collisions.

Connection between ionization coefficient and ionization cross section

The mean free path for the ionisation collisions, λ_{ion} is given by

$$\lambda_{ion} = \frac{1}{n\sigma_{ion}} \qquad \alpha = \frac{1}{\lambda_{ion}}$$

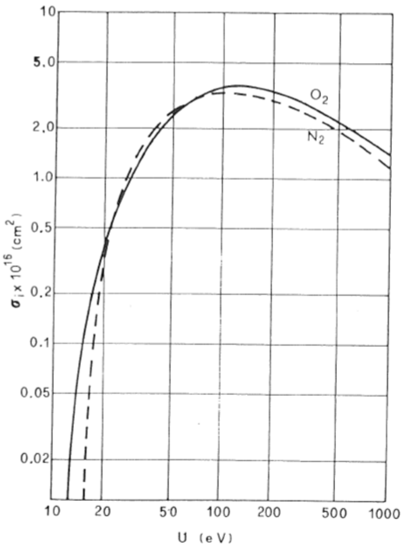
where *n* is the concentration of gas atoms and σ_{ion} is the microscopic cross section for ionisation. Thus the number of ionisation collisions made by an electron in moving a unit length, α, is given by

$$\alpha = n\sigma_{ion}$$

It is observed that

$$\frac{\alpha}{p} = f(E/p)$$

Ionization cross section of electrons

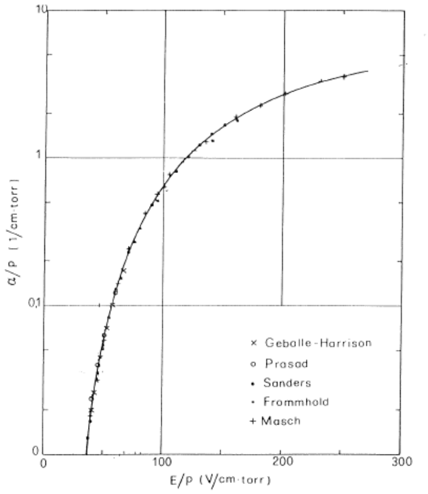


Notes

Even though the electron energy is larger than the ionisation energy this does not mean that the electron will ionise every time it collides with an atom.

The peak is reached around 100 eV.

Variation of ionization coefficient with electric field



You will understand later why the plot is given as *α/p* vs *E/p*.

Photo ionisation

Ionisation of an atom can be caused not only by energetic material particles but also by photons if the photon energy is larger than the ionisation energy of the atom. The process can be represented by the equation

$$A + h\nu = A^{+} + e$$

where A is the target atom, h is the Plank constant and $h\nu$ is the energy of the incident photon.

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Multi photon ionization

The threshold frequency for photo ionisation is given by

$$\nu_n = V_i / h$$

where V_i is the ionisation energy of the atom.

Experiments show that ionisation occurs even if the frequency of the incident photons is below this threshold.

The reason for this is the stepwise ionisation of the atoms where many photons act on the atom simultaneously. Stepwise ionisation caused by many photons is important in ionisation of gases by lasers where in many cases the energy of individual photons is less than the ionisation energy of the target atom.

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Thermal ionisation

With increasing gas temperature the number of atoms that have energies sufficient to cause ionisation increases and so does the number of ionisation collisions.

The Saha equation

In a gas volume heated to a high temperature there are electrons, neutral atoms, and ions.

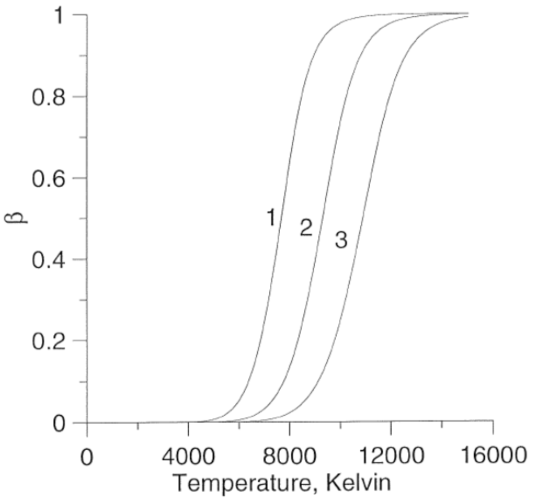
In the mathematical development Saha assumed that all these species are in thermal equilibrium at the temperature of the gas volume.

According to Saha the number of ionised particles in a volume of gas which is in thermodynamic equilibrium is given by

$$\frac{\beta^2}{1-\beta^2} = \frac{2.4x10^{-4}}{p} T^{2.5} e^{-V_i / kT}$$

with $\beta = n_i/n$ where n is the total number of particles and n_i is the number of ions in the volume of gas under consideration.

Effect of temperature on ionization



The data show that the thermal ionisation in air is significant only at temperatures above about 4000 K.

1: V_i = 10 eV
2: V_i = 12.5 eV
3: V_i = 15 eV

Meta-stable excited atoms

Usually, an excited atom in a gas returns from its excited state to the ground state within about10^{-8} sec. releasing the energy as one or more quanta of radiation. These are called normal excited states of the atom.

Metastable excited states are exceptions to this and have relatively long lifetimes: around 1 ms or more.

Ionization by Meat-stable excited atoms

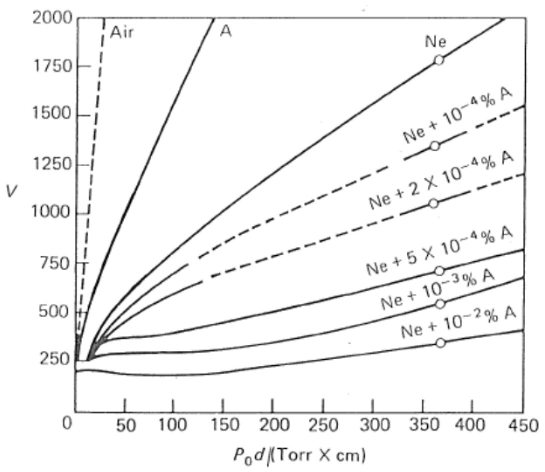
If the metastable excited state has energy equal or slightly higher than the ionisation energy of atoms in the ensemble the process leads to enhanced ionisation. This process is sometimes called the Penning ionisation.

A typical collision process of this kind is given by

$$Ne_m + Ar = Ne + Ar^+ + e$$

where Ne_m is a meta-stable neon atom.

Electrical Breakdown voltage of Neon-Argon Mixture



Note the reduction of breakdown voltage of the mixture in comparison to the virgin gases. The reduction in the breakdown voltage is caused by the meta-stable ionisation.

Ionisation due to positive ions

Using basic laws of mechanics one can easily show that a positive ion needs twice the energy needed by an electron to ionise an atom.

However, the experimental data indicate that positive ions need more energy than this threshold before ionisation from them could be detected.

This may also depend on the fact that the collision is not point like, as in the case of electrons, and the collision energy is distributed in the electron cloud so that it is not concentrated on a single electron.

De-Ionisation processes

Electron-ion recombination

In a volume of gas in which an electrical discharge takes place there is a high concentration of electrons and positive ions. Whenever these oppositely charged particles come closer in collisions they have a tendency to recombine.

The recombination between an electron and an ion can take place in several modes depending on the way the extra kinetic energy of the electron is removed.

Modes of recombination

Radiative recombination

$X^{+} + e = X^{*} + h\nu$

Dissociative recombination

$XY^{+} + e \Leftrightarrow (XY)^{*}_{unstable} \Leftrightarrow X^{*} + Y + (kinetic\ \ energy)$

Three body recombination

$X^{+} + e = \quad X^{*} +$ Kinetic energy of a third body

Notes on recombination

In the case of dissociative recombination the removal of energy to vibrational levels can be done very quickly (order of a vibrational period) and, therefore, high rates of recombination are realised.

Due to the absence of dissociative recombination, the recombination process in mono atomic gases is one or two orders of magnitude slower than in molecular gases.

The probability of three body recombination is rather low at low gas pressures and it increases with increasing pressure.

The probability of recombination decreases as the energy of the electron increases.

Other processes that can influence the process of ionisation

Electron attachment

Some molecules and atoms have an affinity to form negative ions. Gases with electron affinity are called electro negative gases.

The ground state energies of negative ions in these cases are slightly lower than the energy of the ground state of the neutral molecules.

The difference in energy, which is released during the formation of negative ions is called the electron affinity of the atoms or molecules. The stability of the negative ion increases with increasing electron affinity.

Electron affinity of various atoms and molecules

Atom or molecule	Electron affinity (eV)
O	1.461
O_2	0.451
O_3	2.103
NO_2	2.273
NO	.026
SF_6	1.05 – 1.5
H	.714

Observe that oxygen molecule is electronegative with an electron affinity of about 0.5 eV.

The electron affinity of SF_6 is 1.0 - 1.5 eV and this large affinity makes this gas a suitable candidate in increasing the breakdown voltage in high voltage equipment.

Different forms of electron attachment

$e + XY = XY^{-} + h\nu$ (radiative attachment)

$e + XY = X + Y^{-}$ (dissociative attachment)

$e + XY + Z = XY^{-} + Z$ (three body attachment)

where *XY* is a molecular species, $h\nu$ is the energy released as radiation and *Z* is an atom or molecule that acts as a third body.

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Attachment frequency

The attachment frequency is a measure of how fast free electrons are disappearing in a given medium due to attachment.

$$n_e = n_o \exp(-\nu_a t)$$

where n_o is the density of free electrons at $t = 0$ and n_e is the density of free electrons at time t.

For thermalized electrons at T = 300 K the value of $\nu_a = 0.9 \times 10^8$ s^{-1}. That is, the lifetime of a free electron in air is about 10^{-8} s.

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Attachment coefficient

The attachment coefficient η denotes the number of attachment events per electron per unit length of travel.

Like the ionization coefficient

$$\frac{\eta}{p} = f(E/p)$$

Electron detachment

Once an electron is attached to an atom or molecule it is necessary to provide the negative ion with a certain amount of energy to remove the attached electron.

photo detachment $XY^- + hv = XY + e$

where hv is the energy of the photon.

associative detachment process. $X^- + Y = XY + e$

collisional detachment $XY^- + Z = XY + Z + e$

Collisional detachment

This process is important under discharge conditions and most probably the first seeding electrons necessary for the initiation of an electrical discharge in air are produced by this process.

For example, the negative O_2^- ions are decomposed by collisions with molecules possessing an energy high enough to detach an electron. Especially effective in this are the vibrationally excited nitrogen molecules.

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The effect of humidity on collisional detachment

The detachment of electrons from negative Oxygen atoms in humid air is much slower than in dry air because the negative ions become hydrated by attaching it to a cluster of water molecules i.e. O_2^- $(H_2O)_n$ (n = 1, 2,3…). The most probable cluster type in atmospheric pressure is $O_2^-(H_2O)_3$.

The detachment energy increases with cluster number and, therefore, it is more difficult to remove an electron from an hydrated ion.

In order to detach an electron from an hydrated ion the latter has to be de-clustered first and finally the electron should be removed from the negative ion.

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Excitation of molecular vibrations

Low energy electrons can actively excite molecular vibrations and this is one of the most important processes that removes energy from free electrons in air.

In nitrogen, molecular vibrations are excited by electrons in the range of energies 1.8 - 3.3 eV.

In moderate values of *E/p* about 3 – 30 V cm^{-1} $torr^{-1}$, electrons spend about 90% - 95% of the gained energy in exciting molecular vibrations in air and Nitrogen.

At higher values of *E/p* excitation of electronic levels and ionisation are the main energy draining processes.

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Relaxation of vibrational energy

Once a molecule is excited vibrationally it will take some time for this energy to convert back to translational or kinetic energy. This process is called Vibrational-Translational (*VT*) relaxation and is denoted by the relaxation time τ .

In dry air at atmospheric pressure τ = 1.7 x 10^{-2} s. In air containing 0.8 x 10^{-5} g cm^{-3} of water the relaxation time decreases to 7 x 10^{-4} s. That is, Water molecules can deactivate the molecular vibrations.

In hot humid air, τ = 8 x 10^{-5} s at 1000 K and it will decrease to10^{-5} s at 2000 K.

This shows that the *VT* relaxation process is ‘self accelerated’. That is, the relaxation produce heat resulting in an increase of temperature of the gas which in turn decreases the relaxation time thus accelerating the relaxation process.

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Diffusion

Diffusion is the process in which gases move from the regions of high concentration to lower concentrations. This reduces the charge density in the source region thus impeding the discharge development.

In one dimension, for example, the coefficient of diffusion is defined in such a way that the number of particles crossing a unit area perpendicular to the x-axis due to diffusion is

$$N_x = -D\frac{\partial n}{\partial x}$$

n is the concentration of particles

Ambipolar Diffusion

A given volume of an electrical discharge contains both electrons and positive ions. Electrons being smaller than the positive ions diffuse faster than the positives from region of high concentrations.

This will lead to a charge separation and the result will be the creation of an electric field. This electric field will accelerate the drift of positive ions but retard the drift of electrons.

At equilibrium, there will be an equilibrium electric field and both electrons and positive ions diffuse at the same rate. Diffusion taking place under these circumstances is called ambipolar diffusion.

Cathode processes

Electrons in a metal

Under normal conditions electrons in a metal are prevented from leaving the metal by electrostatic forces between the electrons and the ions in the metal lattice.

There are several ways by which Electrons can be removed from the metal.

i) Giving the electrons sufficient kinetic energy to surmount the potential barrier, or the work function.

ii) Reducing the height of the barrier so that the electrons can overcome it.

iii) Reducing the thickness of the barrier so that the electrons can tunnel through it.

Various processes through which one can remove electrons from metals

By the application of heat to the electrode:
Thermionic emission

Through impact of photons on the surface of the electrode:
Photoelectric emission

The incidence of particles such as other electrons, positive ions, neutral molecules, meta-stable atoms on the electrode.

The reduction in the potential barrier height by application of an electric field in the correct direction: Schottky emission

Reducing the thickness of the barrier by the application of an electric field in the correct direction: Field emission

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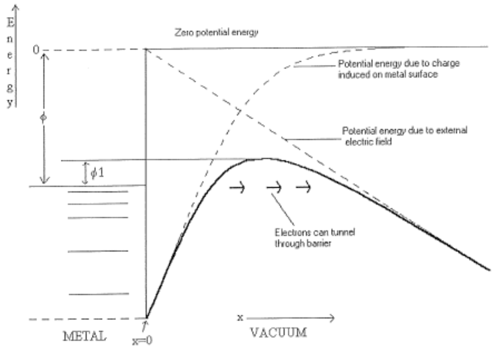
Field emission

Experiments show that at a given temperature with high electric fields the electron emission current is larger than that predicted by Schottky effect.



The illustration shows that the application of the electric field not only reduce the height of the barrier but it will also decrease the thickness of the barrier.

In the absence of the electric field the barrier is infinitely thick but its thickness decreases with increasing electric field.

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Field emission in practice

Experiments show that an appreciable electric field dependent emission current can be obtained for electric fields one to two orders of magnitude smaller than the theoretically estimated critical values in the presence of surface contamination.

The reason for this is that surface contamination causes a reduction in the width of the barrier thus enhancing the field emission process.

If there are protrusions on the surface the electric field at the tip of these protrusions can reach very high values leading to field emission from them.

The field emission process is very important in providing initiatory electrons in the creation of electrical discharges.

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Incidence of positive ions

Elementary particles incident on the surface can cause electrons to be ejected from the surface.

This is a common situation at the surface of the cathode of an electrical discharge where positive ions, having sufficient energy to support an electron to overcome the potential barrier, are incident on the cathode and liberate electrons from it.

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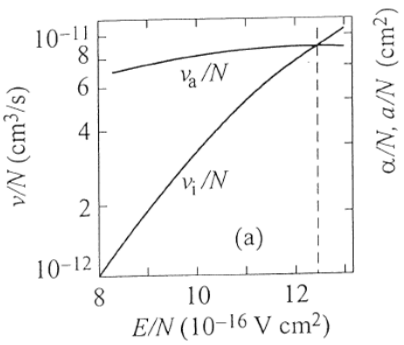
Electrical breakdown

Electrical breakdown

In a given environment with a background electric field there is a competition between the ionisation and deionisation processes. The ionisation processes attempt to increase the number of free electrons in the environment whereas the deionisation processes (electron attachment) attempt to reduce their number.

The relative efficiency of the two competing processes depends on the magnitude of the background electric field.

Critical electric field necessary for Electrical breakdown



Note that the two curves cross around 2.6 x 10^4 V/cm.

Cumulative ionisation is possible only if the background electric field exceeds this critical value.

This value is called the breakdown electric field in atmospheric air and in qualitative analysis it is assumed to be 3.0 x 10^4 V/cm.

Background electric field necessary to cause breakdown in discharge gaps

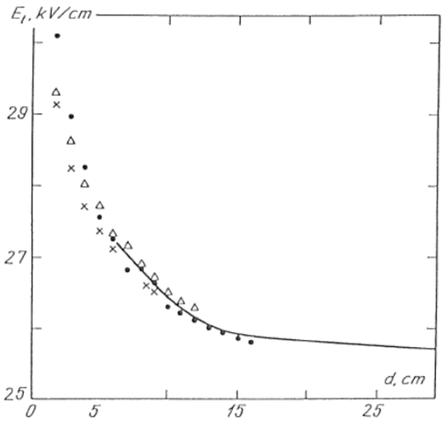


Figure illustrates how the background electric field necessary to cause electrical breakdown in a plane parallel gap (i.e. the electric field is uniform) vary with the plate separation .

Note that for small gap separations the breakdown electric field is larger than the critical field given in the previous slide.

However, the breakdown electric field approaches this critical electric field with increasing gap distance.

Conditions necessary for electrical breakdown

The data shown in the previous slide show that in order to create electrical breakdown two conditions should be satisfied.

First, the electric field in the gap should exceed a critical value.

Second, depending on the magnitude of the electric field there is a certain critical length over which the electric field should extend. This critical length decreases with increasing electric field.

Townsend’s mechanism

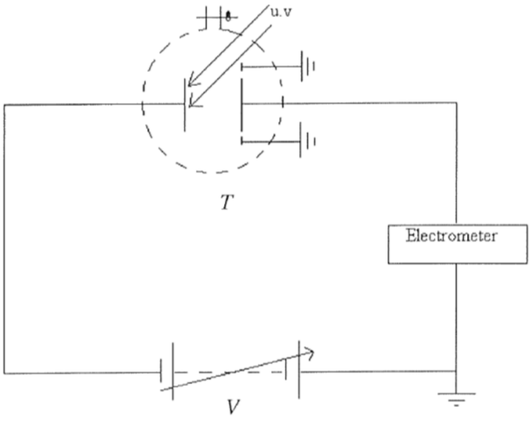
Breakdown in the absence of space charge effects

Townsend’s experiment

The cathode in a low pressure discharge tube filled with an inert gas is illuminated by UV radiation.

A voltage is applied between the cathode and anode.

The current is measured as a function of applied voltage.



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Experimental results (at steady state)

Different stages of the discharge current at steady state:

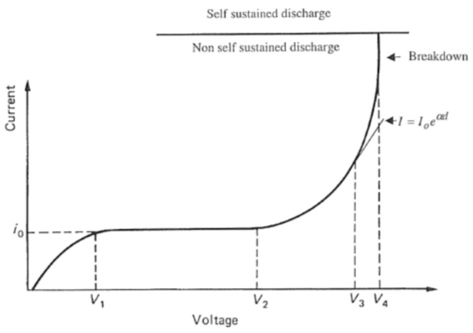
1. Initial increase in current. (Electron Collection efficiency increases)

2. Current remains constant. (All the electrons reach the anode)

3. Current increases exponentially. (Primary Ionization sets in)

4. Current increases faster than exponential. (Secondary ionization set in)

5. Discharge becomes self sustained.



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Primary ionisation stage

At this stage electrons start making ionization collisions. Let α be the number of ionization collisions made by an electron moving a unit length.

Let n be the number of electron moving towards the anode at distance x.
After moving a distance of *dx* the number of electrons increases by *dn*

$$dn = n\alpha dx$$

If the number of electrons leaving the cathode per second is n_0 and the number reaching the anode is n_d, the solution of the above equation (neglecting electron attachment) is

$$n_d = n_o e^{\alpha d}$$

Consequently, The current inside the tube is given by

$$I_d = I_o e^{\alpha d}$$

Note that the discharge is not self sustained. That is, it needs the support of the external agency for its continuation.

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Mathematical explanation of secondary ionization stage

Townsend assumed that at this stage positive ions striking the cathode have enough energy to liberate electrons from the cathode. Let γ be the number of electrons liberated by a positive ion from the cathode.

Let n_+ be the number of electrons liberated from the cathode per second by positive ion bombardment. Then the number of electrons reaching the anode at steady state per second is

$$n = (n_o + n_+)e^{\alpha d}$$

The number of positive ions created per second by this electron flow is

$$n_p = n - (n_o + n_+)$$

Thus

$$n_+ = \{n - (n_o - n_+)\}\gamma$$

Note that the current is not self-sustained

Substituting this in the first Equation we obtain

$$n = \frac{n_o e^{\alpha d}}{1 - \gamma\left(e^{\alpha d} - 1\right)}$$ leading to $$I = \frac{I_o e^{\alpha d}}{1 - \gamma\left(e^{\alpha d} - 1\right)}$$

γ is the Townsend's secondary ionisation coefficient.

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The true picture

In addition to positive ion bombardment there are other physical processes taking place inside the discharge tube that increase the number of electrons, for example

Ionization by positive ions (not that efficient)

Photo emission from the cathode

Ionization of the gas by photons

Incident of metastable ions on the cathode

Irrespective of the secondary ionisation process under consideration the final expression for the current has the same form. Indeed one can include all of them in a single formula as follows:

$$i = i_o \frac{e^{\alpha d}}{1 - \gamma_i\left[e^{\alpha d} - 1\right]}$$

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Townsend's electrical Breakdown criterion

From the previous equation one can see that the current goes to infinity when

$$1-\gamma_i\left[\left(e^{\alpha d}-1\right)\right]=0$$

This condition is known as Townsend's breakdown criterion. This is the condition necessary for a self-sustained electrical discharge.

Note that one electron liberated from the cathode will generate enough secondary effects to generate another electron from the cathode.

One can write this condition also as $\alpha d=\ln(1+\frac{1}{\gamma_i})$

Note that the parameter $\ln(1+\frac{1}{\gamma_i})$

does not change too much and is on the order of 8 – 10 in a Townsend's discharge.

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Townsend breakdown in the presence of attachment

Remember that in deriving the Townsend's equations we have neglected the attachment of electrons to atoms and molecules.

If the attachment is included in the calculations one will end up with the following equation (I leave it as an exercise for you to derive this)

$$I=I_0\frac{\left[\frac{\alpha}{\alpha-\eta}e^{(\alpha-\eta)}-\frac{\eta}{\alpha-\eta}\right]}{1-\gamma\frac{\alpha}{\alpha-\eta}\left[e^{(\alpha-\eta)d}-1\right]}$$

In this equation, η is the number of attachments produced in a path of a single electron travelling a unit length in the direction of the field. Like α, η also depends on *E/p*.

Townsend breakdown in the presence of attachment

Breakdown criterion is

$$\gamma \frac{\alpha}{\alpha-\eta}\left[e^{(\alpha-\eta)d}-1\right]=1$$

If $\alpha > \eta$ breakdown is always possible irrespective of the values of α, η and γ provided that *d* is large enough.

If $\eta > \alpha$ you cannot satisfy the above equation and no breakdown takes place.

The limiting condition at which you will get breakdown is when $\alpha = \eta$.

Townsend's derivation of α

1) Let us represent the ionisation energy of the gas atoms as eV_i .

2) Assume that the energy gained by an electron over a free path of length x is equal to Eex (this is not exactly true, Why?). Note that E is the electric field and e is the electronic charge.

3) Assume that an electron will ionise an atom if it gains energy equal to eV_i during its free path (not exactly true, why?).

4) Therefore, the minimum length of free path, say x_i, which is capable of giving enough energy to the electron so that it can ionise in the next collision is given by

$$eEx_i = eV_i$$

5) Now, from the distribution of free paths which we have obtained earlier, the probability that an electron having a free path larger than x_i is given by

$$P_i = e^{-x_i/\lambda}$$

where λ is the mean free path.

Townsend’s derivation of α

6) The average number of free paths in a unit length is equal to $1/\lambda$.

7) Therefore, the number of ionisation collisions per unit length is

$$\alpha = (1/\lambda)e^{-x_i/\lambda}$$

If we substitute for x_i we find that

$$\alpha = (1/\lambda)e^{-V_i/E\lambda}$$

8) Remember that $(1/\lambda) = A\,p$. Where A is a constant and p is the pressure. Furthermore, let us write $B = A\,V_i$. Substituting these parameters we find that

$$\frac{\alpha}{p} = Ae^{-B/(E/p)}$$

Note that α/p depends only on E/p.

Paschen’s law

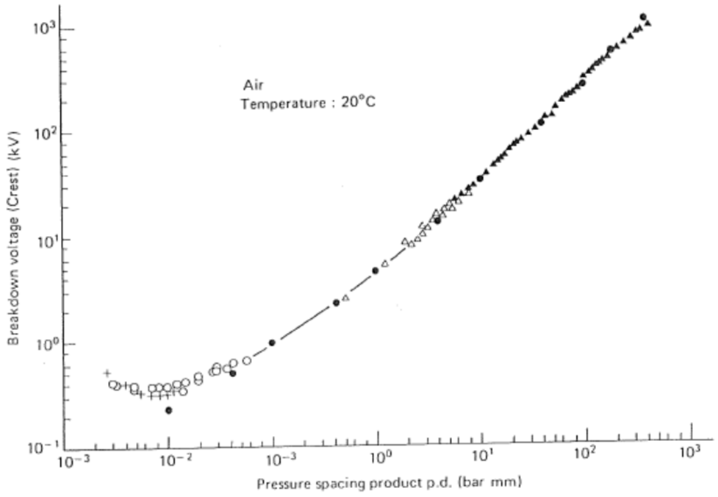
If a slowly increasing voltage is applied across two plane parallel electrodes the electrical breakdown of the gap occurs at a certain critical voltage.

The experimental data show that the breakdown voltage, V_s, is only a function of the gas pressure, p (or gas density), multiplied by the gap length, d. That is

$$V_s = f(pd)$$

This is known as the Paschen’s law.

Paschen’s law experimental data



Derivation of Paschen’s law

The Townsend’s breakdown criterion for a uniform gap of length *d* is given by

$$\alpha d = \ln\left\{\frac{1}{\gamma} - 1\right\} = K$$

Substituting the expression for α

$$Ape^{-B/(V_s/pd)}d = K$$

where V_s is the voltage at which electrical breakdown is observed. Note that in deriving this equation we have used $E = V_S / d$.

Rearranging the above equation we find that

$$V_s = \frac{Bpd}{\ln\left(\frac{Apd}{K}\right)}$$

This equation shows that V_s is a function of *pd*. The general shape of this equation is in agreement with the Paschen’s curve.

Conditions under which Townsend's mechanism is active

Townsend's mechanism is valid when the product of the pressure and the electrode spacing in plane uniform gaps does not exceed about 1 – 2 bar.cm.

Below this limit the space charge of the avalanche is not large enough to change the background field significantly. Under such conditions the breakdown takes place according to the Townsend mechanism.

Streamer mechanism

Electron avalanche

Consider a free electron originated at $x = 0$ in space and moving under the influence of a background electric field directed in the x direction.

Consider an elementary length of width dx located at a distance x from the origin. In travelling across the length dx, n_x number of electrons will give rise to dn additional electrons

$$dn = n_x(\alpha - \eta)dx \qquad n_x = n_0 \Rightarrow x = 0$$

The solution of this equation is

$$n_x = n_o e^{(\alpha - \eta)x}$$

This exponential growth of electrons with distance is called an electron avalanche.

The equation also shows that cumulative ionisation is possible only if $(\alpha - \eta) > 0$. The quantity ($\alpha - \eta$) is known as the effective ionisation coefficient and denoted by $\overline{\alpha}$.

Electron avalanche

From Cloud Chamber photographs

The radius of the avalanche head

As the electron avalanche advances, its tip is spreading laterally by the random diffusion of the electrons.

The average radial distance of diffusion can be calculated from the equation

$$r = \sqrt{4Dt}$$

where $t = x / v_d$ is the time of advance of the avalanche, *D* is the coefficient of diffusion and v_d is the drift velocity of the electrons.

The space charge electric field due to an avalanche

The electric field at the head of the avalanche is given by

$$E_r = \frac{e\{\exp(\bar{\alpha}x)\}}{4\pi\varepsilon_0 r^2}$$

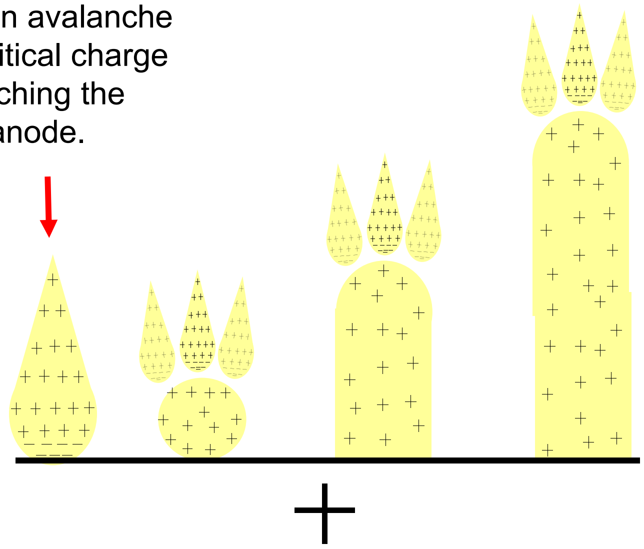
Substituting for *r*

$$E_r = \frac{e\{\exp(\bar{\alpha}x)\}}{4\pi\varepsilon_0}\left(\frac{v_d}{4\pi Dx}\right)$$

This equation shows that with increasing avalanche length the electric field created by the space charge increases and at a certain critical length the electric field generated by the space charge becomes comparable to the background electric field. At this stage an electron avalanche will convert itself to a streamer discharge.

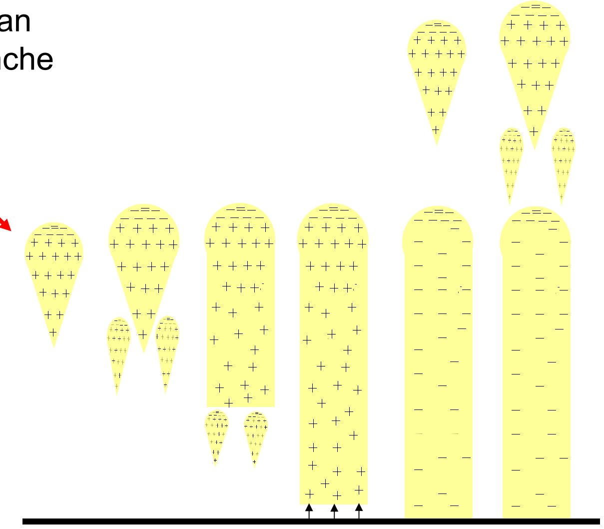
Formation of a positive streamer

Electron avalanche
with critical charge
reaching the
anode.



Formation of a negative streamer

Formation of an
electron avalanche
with critical
charge.



N_c

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Avalanche to streamer transition

The avalanche to streamer transition takes place when the number of charged particles at the avalanche head exceeds a critical value, N_c .

From cloud chamber photographs (and also by calculations) it is found that this critical value is about 10^8.

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Critical avalanche length

The critical avalanche length for transition to a streamer is given by

$$e^{\overline{\alpha} x_c} = 10^8$$

or

$$\overline{\alpha} x_c \approx 18$$

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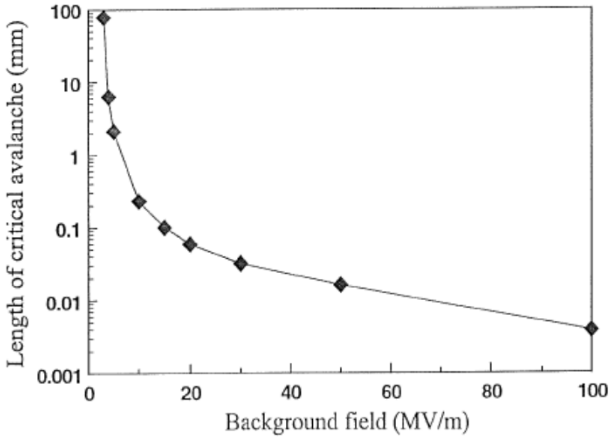
The length of the critical avalanche



Note that the critical avalanche length decreases with increasing electric field.

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Characteristics of streamers

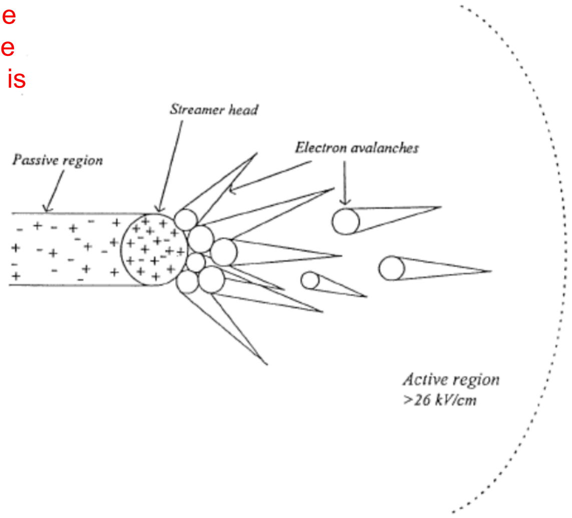
The physical processes taking place at the streamer head and its propagation

- The advancement of the streamer in a given background electric field is based on the distortion of the electric field at the head and the enhanced production of the photons from the head.

- The photons create secondary electrons in front of the streamer head and these secondary electrons give rise to secondary avalanches that will move, in the case of positive streamers, towards the streamer head.

Streamer tip and the active region

Size of the active region (i.e. the region where the electric field exceeds the breakdown electric field) is about 0.2 mm.



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Propagation of positive versus negative streamers

Negative streamers
- Cathode has to supply the necessary electrons.
- Electrons move into low field region leading to formation of negative space charge.

Positive streamers
- Anode has to absorb the electrons
- Electrons move into high field region

These features make the critical electric field necessary for the propagation of positive streamers lower than the negative streamers.

Positive: 5 x 10^5 V/m Negative: (1 -2) x 10^6 V/m

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Physical properties of the streamer channel

The streamer channel consists of a quasi-neutral plasma with an excess of positive or negative charge and the gas in the streamer channel is at ambient temperature.

The streamer radius was found to be on the order of 50 - 100 μm.

The free electron density per unit length of the streamer channel lies in the range of (0.7 – 6) x 10^{12} /cm.

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Parameters that affect the critical background field necessary for streamer propagation

Any variation in the electron loss processes (i.e. electron attachment) can change this electric field.

In air the critical electric field for positive streamer propagation grows from $4.7x10^3$ V/cm at humidity of $3/gm^3$ to $5.6x10^3$ V/cm at $18g/cm^3$.

The critical electric field necessary for streamer propagation also change with the temperature and density of air. For example, the critical electric field decreases with decreasing pressure.

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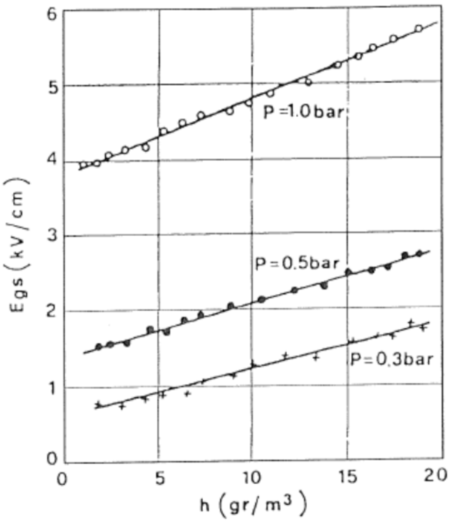
Critical field as a function of pressure



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Streamer speed

The results show that the streamer speed increases with increasing background electric field.

The streamer speed may lie in the ball park $5x10^5$ – $5x10^6$ m/s. The speeds of negative streamers are less than the speed of positive streamers.

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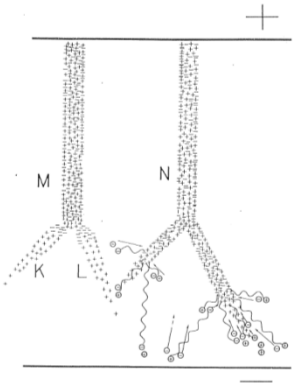
Streamer branching: angle

A propagating streamer may branch frequently (over mm to cm distances). The typical branching angle is about 30°.



Potential gradient of the streamer channel

Experiments conducted with long sparks with lightning impulses show that the average potential gradient of the electrode gap when the positive streamers bridges the gap between the two electrodes is about 5x10^5 V/m.

This indicates that the potential gradient of the positive streamer channels in air at atmospheric pressure is close to this value. Note that this value is approximately the same as the critical electric field necessary for the propagation of positive streamers.

Similar analysis of negative long spark breakdown shows that the potential gradient along negative streamer channels is about (1-2)x10^6 V/m

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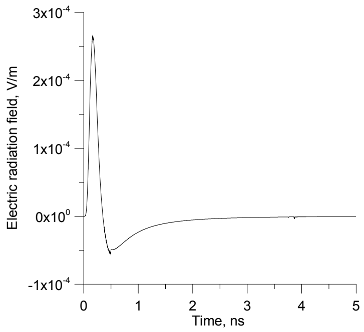
Radiation field from an electron avalanche

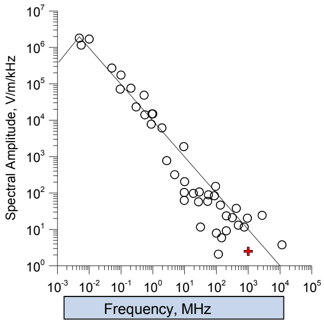


The avalanche is initiated in front of the head of a 2 cm long streamer channel when the background electric field is equal to 575 kV/m.

The radiation field generated by a critical avalanche at a distance of 10 m.

Cooray, V. and Cooray, G., Electromagnetic radiation field of an electron avalanche, Atmospheric Res., 117, 18 -27, 2012.

Contribution from electron avalanches to lightning spectrum

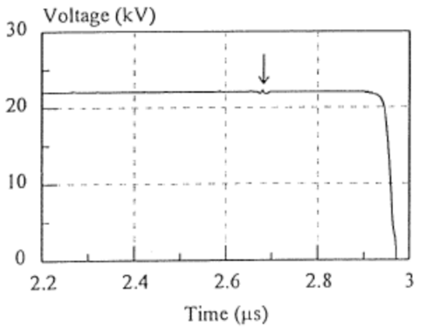


The spectral characteristics of close lightning flashes normalized to a common distance of 10 km. The data points correspond to different measurements and the solid line shows the overall behavior. The diagram is adopted from Pierce [1977] and individual measurements are referenced in Oetzel and Pierce [1969]. The cross represents the predicted contribution from avalanches.

Streamer to spark transition

The streamer is a cold discharge (i.e. the gas temperature in the channel is close to ambient) and the conductivity of the streamer channel is rather small.

Thus, the propagation of the streamer from one electrode to another is not a guarantee that it will result in electrical break down.



Observe that the streamer inception and its subsequent crossing of the gap does not lead to the collapse of the voltage. The streamer channel has to be heated up before full breakdown materialise in the gap.

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Thermalisation

The transition of air in a discharge channel from ambient to a high temperature is called the 'Thermalization' of the discharge.

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Thermalisation of a discharge – stage 1

In the streamer phase (or cold phase) of the discharge many free electrons are lost due to attachment to electronegative Oxygen.

Furthermore, a considerable amount of energy gained by electrons from the electric field is used in exciting molecular vibrations.

Since the electrons can transfer only a small fraction of their energy to neutral atoms during elastic collisions the electrons have a higher temperature than the neutrals. That is, the gas and the electrons are not in thermal equilibrium.

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Themalisation of a discharge – stage 2

As the gas temperature rises to about 1600 – 2000 K rapid detachment of the electrons from Oxygen negative ions supply the discharge with a copious amount of electrons thus enhancing the ionisation.

As the temperature rises the VT relaxation time decreases and the vibrational energy converts back to translational energy thus accelerating the heating process.

As the ionisation process continues the electron density in the channel continue to increase. When the electron density increases to about 10^{17} cm^{-3} a new process starts in the discharge channel.

This is the strong interaction of electrons with other electrons and positive ions through long range Coulomb forces.

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Themalisation of the discharge – stage 3

The Coulomb interaction leads to a rapid transfer of the energy of electrons to positive ions causing the electron temperature to decrease.

The positive ions, having the same mass as the neutrals, transfer their energy very quickly, in a time on the order of 10^{-8} s, to neutrals. This results in a rapid heating of the gas.

At this stage the thermal ionisation sets in causing a rapid increase in the ionisation and the conductivity of the channel. This process is called thermalisation.

The rapid increase in the conductivity of the channel during thermalisation leads to an increase in the current in the discharge channel and the collapse of the applied voltage leading to a spark.

During thermalisation as the electron temperature decreases the gas temperature increases and very quickly all the components of the discharge namely, electrons, ions and neutrals, will achieve the same temperature and the discharge will reach local thermodynamic equilibrium.

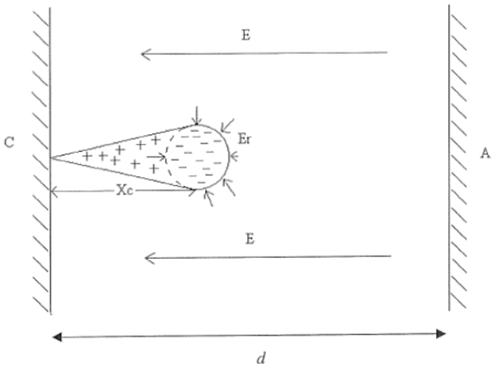
Electrical breakdown criterion in the presence of streamer discharges

As discussed before creation of a streamer in a discharge gap does not necessarily mean that it will always lead to electrical breakdown of the gap.

However, if electrical breakdown does not materialise after the streamers has bridged the gap only a slight increase in the voltage will lead to final breakdown.

Thus, the voltage necessary for the inception of a streamer and the subsequent propagation across the gap can be used as a criterion for electrical breakdown.

Plane uniform gap



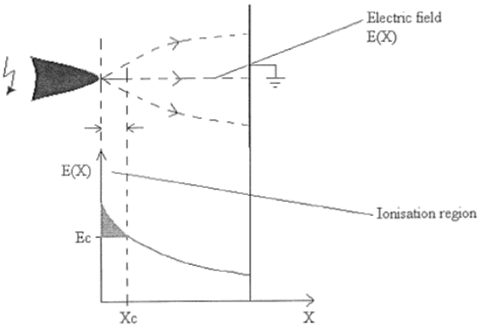
The electrical breakdown criterion could be stated as

$$\bar{\alpha} d \geq 18$$

Note that in order for a streamer to be initiated in the gap the electric field should increase beyond 2.6×10^6 V/m.

Thus, once a streamer is initiated in the gap the conditions necessary for its propagation is already fulfilled in the space between the two electrodes.

Non uniform gap

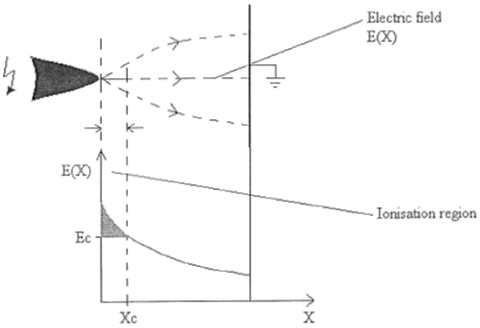


inception criterion: The criterion for the inception of the streamer can be written as

$$\int^{x_c} \overline{\alpha}(x)dx \geq 18$$

where x_c is the axial length of the region within which the electric field is high than 2.6 x 10^4 V/cm.

Non uniform gap



propagation criterion: Once the streamer is created, the background electric field must be able to sustain the streamer propagation.

This explains why it is easier to cause breakdown in a rod-plane gap when the rod is at a positive polarity than when it is at a negative polarity.

Dependence of electrical breakdown conditions on atmospheric conditions

The critical electric field necessary for cumulative ionisation in air, and hence the breakdown voltage of air gaps depends on the density of air.

Let E_c (= 2.6 x 10^4 V/cm) be the critical electric field in air for cumulative ionisation at standard atmospheric conditions (i.e. p_0=1.013 bar, T_0 = 293 K).

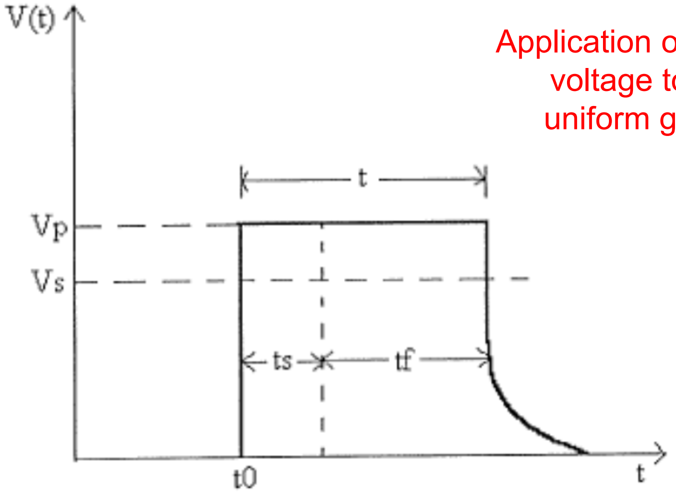
The corresponding critical electric field, E , at non standard atmospheric conditions corresponding to p and T can be obtained from

$$E = E_c \delta \qquad \delta = \left(\frac{pT_o}{p_o T} \right)$$

Statistical nature of electrical breakdown

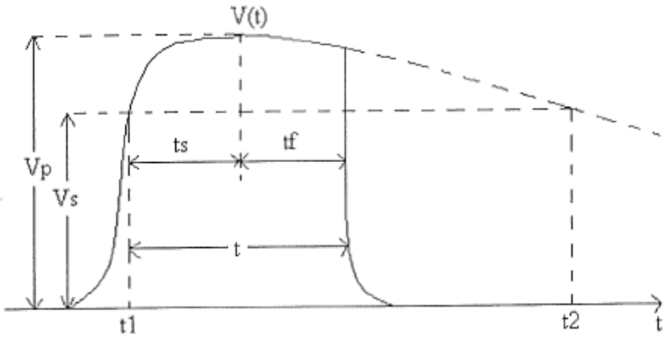
We studied what happens if a uniform DC voltage is applied to a uniform gap. What happens if we apply a transient voltage?

Statistical nature of electrical breakdown



Application of a DC voltage to a uniform gap.

Electrical breakdown under the application of impulse voltages



t_s : Time necessary to find the first electron – Statistical time lag.
t_f : Time necessary for the breakdown process to take place – Formative time lag.

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Statistical nature of the electrical breakdown

Due to the statistical nature of the time lags, when a given number of identical voltage impulses with magnitude exceeding the *static breakdown voltage* V_s is applied to a gap only a certain percentage will lead to breakdown. Thus, for a given voltage impulse there is a certain probability that the gap will breakdown.

V_{b-100} represents 100% breakdown voltage.

V_{b-50} represents the peak voltage in which one half of the applied voltage impulses at this level leads to breakdown.

V_{b-0} represents the largest peak impulse voltage that does not lead to breakdown. It is known as *the impulse withstand level* of the gap.

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The long spark

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The long spark

In small gaps the transformation of the streamer to spark channel takes place directly after the streamer has crossed the gap and reach the grounded electrode.

In the case of long gaps the processes that lead to the electrical breakdown is little bit more complicated. The first phase of the discharge development is a corona discharge (called *first corona*) which takes the form of a burst of filamentary channels from the high voltage electrode.

The next stage is the development of a highly conducting discharge channel called the *leader* from the high voltage electrode.

In the third phase the leader extends, with *the* aid of corona discharges emanating from its head, towards the grounded electrode.

The *final jump* is the last stage of the leader before final breakdown. The final jump starts when the corona streamers emanating from the leader head reaches the grounded electrode.

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Streamer to leader transition in long gaps

Many of the streamers in the first corona have their origin in a common channel called the streamer stem. The streamers stop when the electric field decreases below the critical value necessary for their propagation.

Each individual streamer is a cold discharge and the current associated with this cannot heat the air sufficiently to make it conducting. However, the combined current of all streamers flowing through the stem causes this common region to heat up increasing the conductivity of the stem.

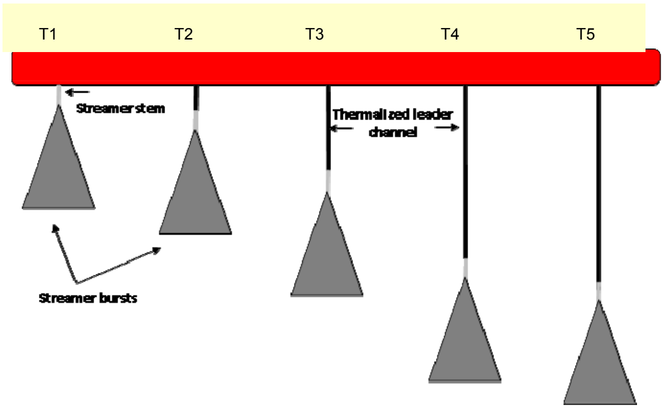
As a result the stem will be transformed into a hot and conducting channel called the leader.

Propagation of positive leader

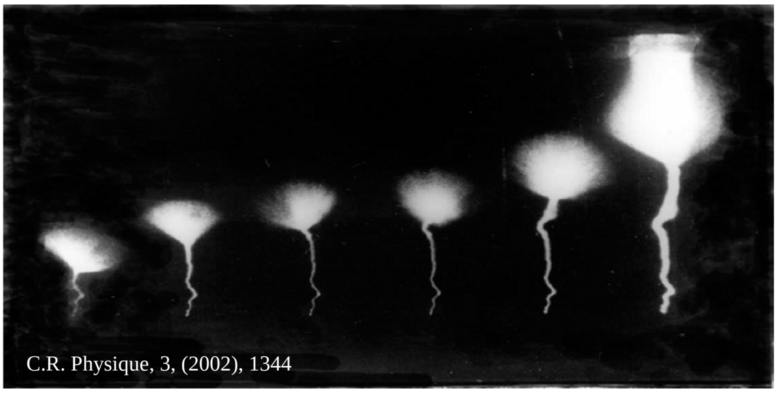
Owing to its high conductivity of the heated stem most of the applied voltage will be transferred to tip of the stem (i.e. the leader channel). The production of streamer discharges now takes place from a common stem located at the head of the leader channel.

With the aid of cumulative streamer currents the new stem gradually transforms itself to a newly created leader section with the streamer process now repeating at the new leader head. The streamer system located in front of the leader is the source of current which heats the air and makes possible the elongation of the leader.

Positive leader propagation (Picture from Introduction to Lightning)



Positive Leader Discharges in the Laboratory



C.R. Physique, 3, (2002), 1344

Streak images of a positive leader propagating in a long air gap

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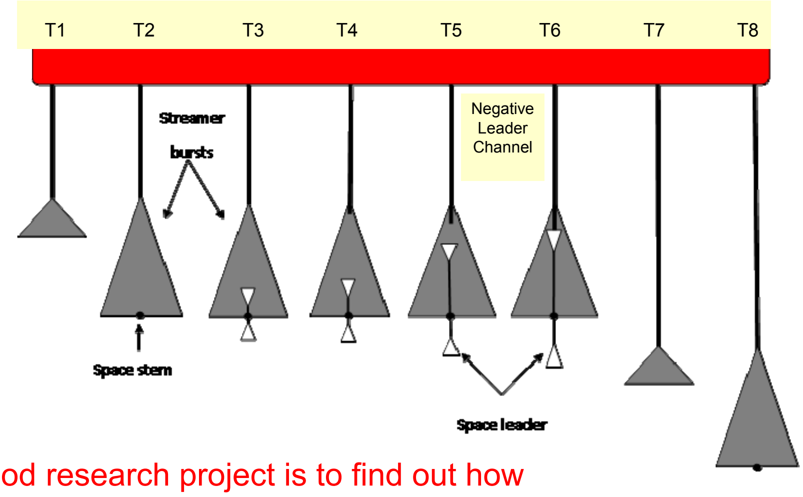
Negative leader propagation

(Picture from Introduction to Lightning)



A good research project is to find out how
the space stem is created.

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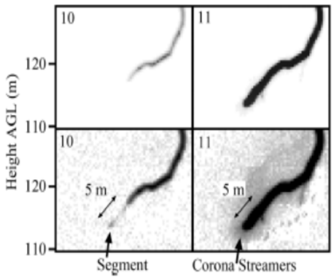
Formation of steps in stepped leaders



Biagi, C.J.; Uman, M.A.; Hill, J.D.; Jordan, D.M.; Rakov, V.A.; Dwyer, J. Observations of stepping mechanisms in a rocket-and-wire triggered lightning flash. J. Geophys. Res. Atmos. **2010**, 115, D23215.

Hill, J.D.; Uman, M.A.; Jordan, D.M. High-speed video observations of a lightning stepped leader. J. Geophys. Res. Atoms. **2011**, 116, D16117.

The formation of a leader step as observed in the negative leader of an altitude triggered lightning. The lower set of diagrams show the contrast enhanced picture of the upper set.

Observe the high luminosity region located ahead of the leader tip in the lower left diagram which is probably the space leader. The lower right diagram shows the newly formed leader step together with the streamers that emanated from the tip of the leader.

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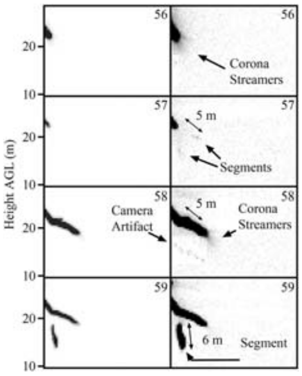
Formation of branches in the negative stepped leader



Time interval between frames is 9.5 µs.

Biagi, C.J.; Uman, M.A.; Hill, J.D.; Jordan, D.M.; Rakov, V.A.; Dwyer, J. Observations of stepping mechanisms in a rocket-and-wire triggered lightning flash. J. Geophys. Res. Atmos. **2010**, 115, D23215.

Hill, J.D.; Uman, M.A.; Jordan, D.M. High-speed video observations of a lightning stepped leader. J. Geophys. Res. Atoms. **2011**, 116, D16117.

Negative leader branching mechanism observed in altitude triggered lightning.

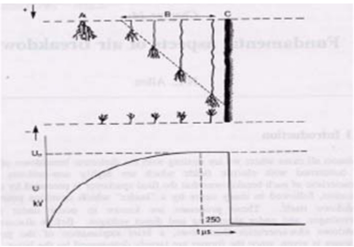
The final jump

As the negative leader comes close to the anode the resulting high electric field at the grounded electrode leads to a burst of positive streamers.

The final jump condition is reached when the negative streamers of the leader meets the positive streamers generated by the grounded electrode.

In the case of lightning, the final jump is reached when the two streamer fronts of negative and connecting leaders meet each other.

Breakdown voltage of long gaps (Positive polarity)



$E_s^+, E_s^-, l_s^+, l_s^-$ are the potential gradients and lengths of positive and negative streamer regions

$E_l^+, E_l^-, l_l^+, l_l^-$ are the potential gradients and lengths of positive and negative leaders

Breakdown voltage is:

$$V_s = E_l^+ l_l^+ + E_s^+ l_s^+ + E_l^- l_l^- + E_s^- l_s^-$$

$$d = l_l^+ + l_s^+ + l_l^- + l_s^-$$

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Rod-Plane Gaps (positive polarity)

In a rod-plane gap we do not have negative streamers, and therefore

$$V_s = E_l^+ l_l^+ + E_s^+ l_s^+$$

Comparison of this equation with the previous one shows that the breakdown voltage is less in this case.

Note also that we can write the above equation as

$$V_s = E_s^+ d - l_l^+ (E_s^+ - E_l^+)$$

Thus the impulse voltages that favour the formation of leaders have a lower breakdown voltage than the ones that generate only streamers in the gap.

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Now we have come to the end.

Hope you have learned something and enjoyed the lectures