Energetic Proton Spectra Measured by the Van Allen Probes

Danny Summers1, Run Shi1,2,3, Mark J. Engebretson4, Kjellmar Oksavik5,6,7,8, Jerry W. Manweiler9, and Donald G. Mitchell9

1Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John’s, Newfoundland, Canada, 2Department of Space Physics, School of Electronic Information, Wuhan University, Wuhan, China, 3Center for Space Physics, Boston University, Boston, MA, USA, 4Department of Physics, Augsburg College, Minneapolis, MN, USA, 5Birkeland Centre for Space Science, Department of Physics and Technology, University of Bergen, Bergen, Norway, 6The University Centre in Svalbard, Longyearbyen, Norway, 7Center for Space Science and Engineering Research, Virginia Tech, Blacksburg, VA, USA, 8Fundamental Technologies LLC, Lawrence, KS, USA, 9Space Department, The Johns Hopkins University Applied Physics Laboratory, Laurel, MD, USA

Abstract We test the hypothesis that pitch angle scattering by electromagnetic ion cyclotron (EMIC) waves can limit ring current proton fluxes. For two chosen magnetic storms, during 17–20 March 2013 and 17–20 March 2015, we measure proton energy spectra in the region 3 ≤ L ≤ 6 using the RBSPICE-B instrument on the Van Allen Probes. The most intense proton spectra are observed to occur during the recovery periods of the respective storms. Using proton precipitation data from the POES (NOAA and MetOp) spacecraft, we deduce that EMIC wave action was prevalent at the times and L-shell locations of the most intense proton spectra. We calculate limiting ring current proton energy spectra from recently developed theory. Comparisons between the observed proton energy spectra and the theoretical limiting spectra show reasonable agreement. We conclude that the measurements of the most intense proton spectra are consistent with self-limiting by EMIC wave scattering.

1. Introduction

Ion pitch angle scattering by electromagnetic ion cyclotron (EMIC) waves is considered to be a significant ring current loss mechanism (e.g., Kennel & Petschek, 1966; Cornwall et al., 1970; Lundblad & Soraas, 1978; Soraas et al., 1999; Erlandson & Ukhorskiy, 2001; Yahmina et al., 2002; Summers, 2005; Fraser et al., 2006; Jordanova et al., 2007; Yahnini & Yahmina, 2007; Engebretson et al., 2008; Sakaguchi et al., 2008; Usanova et al., 2010; Spasojevic et al., 2011; Xiao et al., 2012; Soraas et al., 2013; Yuan et al., 2014, 2016; Xiong et al., 2016). In this paper, we revisit the concept of the limitation of ring current ion flux by EMIC waves. Kennel and Petschek (1966) originally suggested that, under certain conditions, electromagnetic wave gain due to convective growth along a magnetic field line would equal the wave energy lost due to wave reflection at the (ionospheric) ends of the field line and that this balance would produce a self-limiting particle flux. This idea was proposed to apply to both EMIC wave-ion interactions and whistler mode wave-electron interactions. Schulz and Davidson (1988) were able to obtain the Kennel-Petschek limiting particle spectrum (J ∝ E1/2) at large energy E in a nonrelativistic regime. Summers et al. (2009), in a relativistic study, replaced the aforementioned wave energy criterion used by Kennel and Petschek (1966) by a condition that specifies the wave gain over a given convective length scale and obtained expressions for the limiting differential and integral electron fluxes at a given L-shell. Mauk and Fox (2010) obtained Kennel-Petschek-type differential flux limits by using flexible analytic fits to measured electron distributions and applied their results to solar system electron radiation belts. Nonlinear wave gain effects were included in the limiting electron flux problem by Summers et al. (2011). Tang and Summers (2012) compared measured average electron differential and integral fluxes at Saturn with corresponding Kennel-Petschek limits over a range of L-shells (4.5 < L < 11). Mauk (2013, 2014) obtained limiting ion spectra moderated by EMIC waves in a cold multi-ion background plasma and compared them with measured planetary ring current spectra.

Summers and Shi (2014) were able, for the first time, to solve the limiting electron flux problem in a relativistic regime without assuming a predetermined form for the particle distribution. They showed that the limiting electron differential flux moderated by whistler mode waves is determined by an integral equation that can be solved by standard numerical techniques. Subsequently, Summers and Shi (2015) likewise showed in a
relativistic regime that the limiting ion differential flux subject to scattering by EMIC waves in a cold multi-ion background plasma is also given by an integral equation that must be solved numerically.

In the present study, we test the validity of the formulation of the limiting ion flux by Summers and Shi (2015) by comparing numerical (theoretical) solutions for the limiting flux with experimental proton flux profiles. Specifically, we measure proton differential flux profiles in the region $3 \leq L \leq 6$ over two chosen magnetic storms using data from the RBSPICE-B instrument on the Van Allen Probes. In section 2 we describe the data analysis associated with RBSPICE-B, and in section 3 we present the proton flux observations obtained. In section 4 we provide a condensed theoretical account of how we determine the limiting proton energy spectrum. Typically, regions of EMIC wave activity are highly localized and hence relatively difficult to observe by the Van Allen Probes. Therefore, as a useful proxy for EMIC wave activity, we use proton precipitation events observed by the POES (NOAA and MetOp) spacecraft. Various studies collectively provide firm evidence of the link between EMIC wave activity and proton precipitation spikes, and we briefly review these studies in section 5. We describe our observations of POES precipitation events in section 6. In section 7 we show comparisons of the observed proton spectra with the theoretical limiting solutions. Finally, in section 8 we provide a brief summary and state our conclusions.

2. Data Analysis

RBSPICE is a time-of-flight versus total energy instrument on board the Van Allen Probes (spacecraft A and B) with a perigee of ~600 km and an apogee of ~5.8 RE (Mauk et al., 2012). The orbital period of the Van Allen Probes is approximately 9 h. RBSPICE measures ions over the energy range from ~20 keV to ~1 MeV, as well as electrons in the range from 25 keV to 1 MeV. The instrument further distinguishes the ion composition. It provides high-resolution measurements of energy and pitch angle for hydrogen (10–600 keV), helium (75–600 keV), and oxygen (40 keV–1 MeV) (Mitchell et al., 2013). In the present investigation, we focus on hydrogen. The proton abundances are from the time-of-flight versus total energy instrument feature (Mitchell et al., 2013) and cover the energy range ~10 keV to ~600 keV. The total energy of a particle that transits the detector system is measured by a solid state detector. Ion velocities are determined by measuring the time-of-flight of a particle through the instrument between collisions using an entrance and exit foil. We utilize the level 3-PAP data that are already pitch angle binned. For the two magnetic storm periods (specified below) that we have chosen to study, RBSPICE-B was operational with its high voltage system fully engaged, covering a wider range of $L$-shells than RBSPICE-A. Therefore, the data from RBSPICE-B are used for this study. The data can be found via the website http://rbspiceb.ftecs.com/Level_3PAP/. To obtain the proton differential flux profile as a function of energy at different $L$-shells, the measured proton differential flux at 90° pitch angle is selected and binned using 0.1 RE bins. The McIlwain $L$-shell parameter is computed for 90° particles using the OP77Q external field (Olson & Pfister, 1982) and IGRF internal field (Finlay et al., 2010).

3. RBSPICE-B Observations

The geomagnetic storm periods considered in this study are as follows:

Case 1: 17–20 March 2013

and

Case 1: 17–20 March 2015

As illustrated in Figure 1, Case 1 comprises Van Allen Probe B orbits 527–535, while Case 2 comprises orbits 2,469–2,477. For $L$-shells of interest in the present work, namely, $3 \leq L \leq 6$, Van Allen Probe B is located basically on the nightside for both Cases 1 and 2. Time profiles of the $Dst$ and $Kp$ indices for Cases 1 and 2 are respectively given in the bottom two panels of Figures 3 and 6.

Case 1 involves a strong storm with minimum $Dst = -132$ nT. The main phase extends from 06:00 UT, 17 March to 21:00 UT, 17 March, and the recovery phase throughout 18–20 March 2013 is considered. Substorm activity, as indicated by $Kp$ levels, is strong during the main phase and weak during the recovery phase.

Case 2 comprises a severe storm with minimum $Dst = -223$ nT. The main phase occurs during 07:30 UT, 17 March to 22:45 UT, 17 March, and the recovery phase during 18–20 March 2015 is considered. Substorm activity is extreme during the main phase and remains strong throughout the recovery phase.
In describing Cases 1 and 2 as respectively “strong” and “severe” storms, we have used the classification of magnetic storms by Loewe and Prölss (1997). We should note here in fact that the physical concept that we are addressing in this paper, namely, self-limiting of trapped particle fluxes by wave scattering, is not necessarily associated only with storms. For instance, this concept is also valid during moderate substorm activity in the absence of magnetic storms.

Figure 2 provides a broad overview of the variation of proton differential flux intensity at energies $E = 55$ keV, $100$ keV, and $600$ keV, with respect to $L$-shell, over the duration of the magnetic storm periods Cases 1 and 2. For Case 1, at $E = 55$ keV, $100$ keV, following initiation of the storm, there is a broad increase in flux over the $L$-shell range $4 \lesssim L \lesssim 5$, which is maintained throughout the recovery phase; at $E = 600$ keV, there is a relatively

**Figure 1.** Trajectories of Van Allen Probe B projected onto the equator for Cases 1 and 2, labeled by color-coded orbit number.

**Figure 2.** Local proton differential flux measured by RBSPICE-B as a function of McIlwain $L$-shell and universal time UT, for Cases 1 and 2, at each of the energies $E = 55$ keV, $100$ keV, and $600$ keV.
Figure 3. Proton spectra measured by RBSPICE-B for Case 1. The spectra are color-coded by orbit number at the indicated $L$-shells. In the bottom panels are time profiles of the Dst and Kp indices. The vertical colored lines indicate the times at apogee for each color-coded orbit.
insignificant increase in flux as a result of the storm, and the flux remains at a low level over all $L$-shells during the storm. For Case 2, at $E = 55$ keV, 100 keV, there is a broad increase in flux over $L$-shells, $3 < L < 4$, due to the storm; at $E = 600$ keV, dependent on $L$-shell, fluxes undergo a net loss or insignificant increase as a result of the storm.

In the upper eight panels of Figure 3 we show proton (differential) spectra for Case 1 at $L = 3, 4, 5, 6$ for outbound and inbound satellite passes, with the spectra color-coded according to orbit number and universal time. In the lower panels showing the time profiles of the $Dst$ and $Kp$ indices, the vertical colored lines indicate times of apogee of RBSP-B. In general, a broad increase in spectral intensity is apparent over the course of the storm over most $L$-shells and energies. In particular, we observe the strong relatively flat spectrum over energies 50 keV–200 keV, at $4 < L < 5$, that is produced over the recovery period. We also observe that at each $L$-shell for which both outbound and inbound spectra are shown in Figure 3, the outbound and inbound spectra show very similar behavior. No proton data are available at $L = 3$ for the outbound passes in Case 1.

Figure 4 shows detailed time variations of proton fluxes for Case 1 over the course of the storm, for inbound passes, at each energy $E = 44.7$ keV, 99.4 keV, 180.5 keV, and 597.6 keV, for the $L$-shells $L = 3, 4, 5, 6$. The colored rectangular region in each panel represents the duration of the main phase of the storm. The strong flux at $E = 44.7$ keV is maintained or increased at each $L$-shell during the storm. We note, in particular, the significant increases in flux, by about an order of magnitude over prestorm values, for energies $E = 99.4$ keV and $E = 180.5$ keV at $L = 4$. At $E = 597.6$ keV the relatively low prestorm flux does not significantly increase at any $L$-shell over the course of the storm. Figure 5 shows detailed time profiles of proton fluxes for Case 1 at the same energies and $L$-shells as Figure 4, but for outbound passes. Overall, Figures 4 and 5 indicate very similar behavior of the proton flux variations over the storm as measured during the inbound and outbound satellite passes.

Figure 6 shows proton spectra for Case 2 in an identical format to that used in Figure 3 for Case 1. Many features of the proton spectra shown in Figures 3 and 6 are similar, including the general intensification of the spectra during the course of the storm. However, the proton spectra in Figure 6, the case of a severe storm, differ from the spectra in Figure 3 in two significant ways. First, strong enhancements in spectra not only occur for $L = 4$ and 5 but also occur at $L = 3$. Second, the range of energies over which the enhanced
spectra are largely “flat” is from 50 keV to approximately 300 keV. Time profiles of the proton fluxes for Case 2 at each of the energies \( E = 44.7 \) keV, 99.4 keV, 180.5 keV, and 597.6 keV for each \( L \)-shell, \( L = 3, 4, 5, \) and 6 are shown in Figure 7 for inbound passes and Figure 8 for outbound passes. We note, in particular, that at \( L = 3 \), the increase in flux for \( E = 99.4 \) keV is by 2 orders of magnitude, and for \( E = 180.5 \) keV, the increase in flux is well in excess of 1 order of magnitude over prestorm values. Even for this severe storm, at \( E = 597.6 \) keV, proton fluxes do not increase significantly at any \( L \)-shell.

4. Calculation of the Limiting Energy Spectrum

We consider field-aligned EMIC waves generated near the Earth’s magnetic equator by a trapped energetic ring current proton population with an anisotropic distribution \( f \) given by

\[
f(p^{\parallel}, p^{\perp}) = (\sin \alpha)^{2s} g(p),
\]

where \( s (>0) \) is the pitch angle index and the function \( g \) describes the energy spectrum; \( p = \sqrt{\gamma^2 + p^{\perp}^2} \), where \( p^{\parallel} = \gamma v^{\parallel}/c \) and \( p^{\perp} = \gamma v^{\perp}/c \) are the normalized parallel and perpendicular components of particle momentum; \( v^{\parallel} \) and \( v^{\perp} \) are the particle velocity components parallel and perpendicular to the background magnetic field, \( \gamma = \sqrt{1 + p^2} \) is the Lorentz factor; \( \alpha = \tan^{-1}(p^{\perp}/p^{\parallel}) \) is the particle pitch angle. The omnidirectional differential proton flux is then

\[
J_x(E) = 4\pi (m_p c^2)^2 \int_0^\gamma \gamma^2 p^2 f(\cos \alpha) = 2\pi^{3/2} \Gamma(s + 1) / \Gamma(s + 3/2) \left( \frac{m_p c}{m_p c^2} \right)^2 \gamma^2 g(p),
\]

where \( \Gamma \) is the gamma function and \( m_p \) is the proton rest mass. The proton kinetic energy \( E \) is given by

\[
E = \left( \sqrt{1 + p^2} - 1 \right) m_p c^2
\]

so that \( p^2 = \left( \frac{E}{m_p c^2} \right) \left( \frac{E}{m_p c^2} + 2 \right) \).

We suppose that the EMIC waves generated by the distribution (1) undergo a specified gain \( G \) in amplitude during propagation over a convective path length \( LR \) along a magnetic field line, where \( L \) denotes magnetic
Figure 6. As in Figure 3, except for Case 2.
Figure 7. Proton fluxes from RBSPICE-B (inbound passes) as a function of universal time UT for Case 2 at the given energies and L-shells. The shaded region represents the storm main phase.

Figure 8. As in Figure 7, except for outbound passes.
shell and $R_E$ is the Earth’s radius. Then, as detailed by Summers and Shi (2015), it can be shown that a limiting energy distribution $g(p) = g^\ast(p)$ can be achieved where $g^\ast$ is determined by the equation,

$$
\int_{X_0}^\infty \left[ 1 - \left( 1 - \frac{\gamma \omega / \Omega_p}{1 - \gamma_0 \omega / \Omega_p} \right)^2 \right]^{1/2} \left[ \frac{s}{\nu} \left( 1 + s \right) \frac{\omega}{\Omega_p} - \frac{\left( 1 + s \right) \omega}{\Omega_p} \left( \frac{\omega}{\Omega_p} \right)^2 \left( 1 - \frac{\gamma_0 \omega / \Omega_p}{1 - \gamma \omega / \Omega_p} \right)^2 \right] g^\ast(p) \, dX, \tag{4}
$$

where $\omega$ is the (real) wave frequency, $X = p^2$, $\gamma = \sqrt{1 + X}$, and $\Omega_p = eB_0/(m_e c)$ is the proton gyrofrequency where $e$ is the electronic charge. We assume that the background magnetic field strength $B_0$ equals the equatorial dipole value, $B_0 = B_0/L^3$. In addition, in equation (4), we have set $X_0 = p_0^2$ and $\gamma_0 = \sqrt{1 + X_0}$, where $p_0$ is the minimum proton momentum for gyroresonance given by

$$
p_0 = \left[ n(\Omega_p/\omega) - \left( \Omega_p/\omega \right)^2 + n^2 - 1 \right] / (n^2 - 1), \tag{5}
$$

where $n = ck/\omega$ is the refractive index and $k$ is the (real) wave number.

We further assume that the background plasma component is cold and contains hydrogen (H$^+$), helium (He$^+$), and oxygen (O$^+$) ions. The cold plasma dispersion relation for $L$-mode EMIC waves can be expressed as

$$
n^2 = 1 - \frac{1}{a_e(\omega/\Omega_p)} \left( \frac{1}{1 + \omega_0/\Omega_p} + \frac{\eta_1}{\omega_0/\Omega_p - 1} + \frac{\eta_2}{4\omega_0/\Omega_p - 1} + \frac{\eta_3}{16\omega_0/\Omega_p - 1} \right), \tag{6}
$$

where $a = |\Omega_p|^2/\omega_0^2$ is a cold plasma parameter, $|\Omega_p| = eB_0/(m_e c)$ is the electron gyrofrequency, and $m_e$ is the electron rest mass; $\omega_0 = (4\pi N_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, and $N_0$ is the background electron number density; $e = m_e/m_p$, $\eta_1 = N_1/N_0$, $\eta_2 = N_2/N_0$, and $\eta_3 = N_3/N_0$, where $N_1$, $N_2$, and $N_3$ are respectively the hydrogen, helium, and oxygen ion number densities. For charge neutrality we require that $\eta_1 + \eta_2 + \eta_3 = 1$.

Equation (4), which is an integral equation for $g^\ast$, is valid for all frequencies $\omega$ over which wave growth occurs. The equation can be readily transformed into a linear Volterra integral equation of the first kind (see Summers & Shi, 2015) and can be solved by standard numerical techniques, as given, for instance, by Press et al. (2007). Once the solution for $g^\ast$ has been found, the limiting omnidirectional differential proton flux (from (2)) is then given by

$$
J^\ast_{4\pi}(\gamma) = 2\pi^{1/2} \frac{\Gamma(s + 1)}{\Gamma(s + 3/2)} \left( m_p c^2 \right)^s \gamma^s g^\ast(p), \tag{7}
$$

where $E = (\sqrt{1 + X} - 1)m_p c^2$.

While analytical solutions of equation (4) are not readily found, Summers and Shi (2015) were able to obtain a useful asymptotic solution for $g^\ast$ valid for large $X$ (or, equivalently, large $E$) of the form $g^\ast \sim 1/X^{3/2}$ as $X \to \infty$. The corresponding asymptotic solution for the limiting energy spectrum $J^\ast_{4\pi}$ given by result (7) is found to be

$$
J^\ast_{4\pi}(E) = \frac{eB_0}{\pi^{1/2} eR_E} \left( \frac{s + 1}{s} \right) \frac{\Gamma(s + 1)}{\Gamma(s + 3/2)} \frac{G}{L^2 E}, \quad E \to \infty \tag{8}
$$

The simple asymptotic solution (8) can be readily compared with observed particle spectra at large energy $E$. Further, solution (8) serves as a useful reference against which numerical solutions of equation (4) can be compared.

5. Relation of POES Observations of Proton Spikes to EMIC Wave Activity: A Brief Review

Literature over several decades has established a strong link between EMIC wave activity and particle precipitation spikes at midlatitudes. In particular, papers by Lundblad and Soraas (1978), Søraas et al. (1980, 1999),
Yahnina et al. (2000, 2003), and Sandanger et al. (2007) all use POES (Polar Operational Environmental Satellites) data (or earlier satellite data with similar instrumentation) to relate midlatitude proton spikes to EMIC wave activity. In the following paragraph we briefly discuss these papers and related studies, in order to place into context our own use (described in section 6) of POES proton precipitation events as evidence of EMIC wave activity.

The anisotropic pitch angle distribution of ring current protons provides free energy for EMIC waves, which leads to pitch angle diffusion, proton precipitation into the atmosphere, and the formation of stable auroral red (SAR) arcs at midlatitudes (Cornwall et al., 1971). Observations of energy spectra and pitch angle distributions have verified that the proton fluxes and pitch angle anisotropies in the ring current are sufficient to cause significant wave growth (Joselyn & Lyons, 1976). The pitch angle scattered ring current protons will precipitate into the midlatitude atmosphere, where low-altitude observations have identified proton precipitation spikes in connection with SAR arcs (Lundblad & Soraas, 1978). Søraas et al. (1999) linked the same midlatitude proton precipitation spikes to EMIC wave-particle interaction. Jordanova et al. (1996) modeled ring current proton precipitation due to EMIC wave scattering and found the maximum precipitation within regions of maximum EMIC wave activity. Yahnina et al. (2000, 2003) related the midlatitude proton precipitation spikes to EMIC wave activity via Pc1 and IPDP pulsations. Sandanger et al. (2007) also found observational evidence that EMIC wave activity can lead to additional scattering loss of relativistic electrons during geomagnetic storms.

| Table 1 |
| Times and Locations of NOAA/MetOp Observations of Precipitating Protons for Case 1 (2013) |

<table>
<thead>
<tr>
<th>Month/d</th>
<th>Time (UT)</th>
<th>MLT</th>
<th>Peak L</th>
<th>Min L</th>
<th>Max L</th>
<th>Amplitude</th>
<th>Spacecraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/18</td>
<td>00:29:00-00:30:00</td>
<td>15.00</td>
<td>3.81</td>
<td>3.79</td>
<td>3.82</td>
<td>6,000</td>
<td>NOAA15</td>
</tr>
<tr>
<td>3/18</td>
<td>01:17:00-01:19:00</td>
<td>15.90</td>
<td>3.97</td>
<td>3.93</td>
<td>4.01</td>
<td>2,000</td>
<td>NOAA19</td>
</tr>
<tr>
<td>3/18</td>
<td>13:12:00-13:13:00</td>
<td>12.90</td>
<td>3.93</td>
<td>3.87</td>
<td>3.98</td>
<td>30,000</td>
<td>NOAA19</td>
</tr>
<tr>
<td>3/18</td>
<td>18:11:00-18:12:00</td>
<td>10.00</td>
<td>4.26</td>
<td>4.12</td>
<td>4.39</td>
<td>200,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/19</td>
<td>03:34:00-03:38:00</td>
<td>6.60</td>
<td>4.28</td>
<td>4.19</td>
<td>4.36</td>
<td>90,000</td>
<td>NOAA16</td>
</tr>
<tr>
<td>3/19</td>
<td>07:03:00-07:05:00</td>
<td>6.00</td>
<td>4.38</td>
<td>4.33</td>
<td>4.42</td>
<td>8,000</td>
<td>NOAA16</td>
</tr>
<tr>
<td>3/19</td>
<td>17:05:00-17:06:00</td>
<td>9.80</td>
<td>3.93</td>
<td>3.85</td>
<td>4.00</td>
<td>60,000</td>
<td>MetOp1</td>
</tr>
<tr>
<td>3/20</td>
<td>05:37:00-05:38:00</td>
<td>7.20</td>
<td>3.93</td>
<td>3.90</td>
<td>3.96</td>
<td>10,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/20</td>
<td>07:20:00-07:21:00</td>
<td>3.20</td>
<td>3.70</td>
<td>3.64</td>
<td>3.76</td>
<td>10,000</td>
<td>NOAA15</td>
</tr>
<tr>
<td>3/20</td>
<td>09:39:00-09:41:00</td>
<td>10.40</td>
<td>3.82</td>
<td>3.78</td>
<td>3.86</td>
<td>8,000</td>
<td>NOAA16</td>
</tr>
</tbody>
</table>

Yahnina et al. (2000, 2003), and Sandanger et al. (2007) all use POES (Polar Operational Environmental Satellites) data (or earlier satellite data with similar instrumentation) to relate midlatitude proton spikes to EMIC wave activity. In the following paragraph we briefly discuss these papers and related studies, in order to place into context our own use (described in section 6) of POES proton precipitation events as evidence of EMIC wave activity.

The anisotropic pitch angle distribution of ring current protons provides free energy for EMIC waves, which leads to pitch angle diffusion, proton precipitation into the atmosphere, and the formation of stable auroral red (SAR) arcs at midlatitudes (Cornwall et al., 1971). Observations of energy spectra and pitch angle distributions have verified that the proton fluxes and pitch angle anisotropies in the ring current are sufficient to cause significant wave growth (Joselyn & Lyons, 1976). The pitch angle scattered ring current protons will precipitate into the midlatitude atmosphere, where low-altitude observations have identified proton precipitation spikes in connection with SAR arcs (Lundblad & Soraas, 1978). Søraas et al. (1999) linked the same midlatitude proton precipitation spikes to EMIC wave-particle interaction. Jordanova et al. (1996) modeled ring current proton precipitation due to EMIC wave scattering and found the maximum precipitation within regions of maximum EMIC wave activity. Yahnina et al. (2000, 2003) related the midlatitude proton precipitation spikes to EMIC wave activity via Pc1 and IPDP pulsations. Sandanger et al. (2007) also found observational evidence that EMIC wave activity can lead to additional scattering loss of relativistic electrons during geomagnetic storms.

| Table 2 |

<table>
<thead>
<tr>
<th>Month/d</th>
<th>Time (UT)</th>
<th>MLT</th>
<th>Peak L</th>
<th>Min L</th>
<th>Max L</th>
<th>Amplitude</th>
<th>Spacecraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/18</td>
<td>23:30:00-23:30:45</td>
<td>21.40</td>
<td>3.16</td>
<td>3.12</td>
<td>3.20</td>
<td>7,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/18</td>
<td>23:30:45-23:31:00</td>
<td>21.30</td>
<td>3.05</td>
<td>3.00</td>
<td>3.10</td>
<td>5,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/19</td>
<td>00:23:30-00:24:30</td>
<td>21.60</td>
<td>2.95</td>
<td>2.92</td>
<td>2.97</td>
<td>2,000</td>
<td>MetOp1</td>
</tr>
<tr>
<td>3/19</td>
<td>05:25:00-05:26:00</td>
<td>3.20</td>
<td>3.10</td>
<td>3.08</td>
<td>3.12</td>
<td>4,000</td>
<td>NOAA15</td>
</tr>
<tr>
<td>3/19</td>
<td>05:54:30-05:56:30</td>
<td>7.80</td>
<td>3.04</td>
<td>3.00</td>
<td>3.10</td>
<td>10,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/19</td>
<td>05:54:30-05:56:30</td>
<td>7.70</td>
<td>3.18</td>
<td>3.16</td>
<td>3.25</td>
<td>10,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/19</td>
<td>07:38:00-07:39:00</td>
<td>7.50</td>
<td>3.20</td>
<td>3.10</td>
<td>3.30</td>
<td>50,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/19</td>
<td>07:38:30-07:39:30</td>
<td>6.40</td>
<td>3.25</td>
<td>3.03</td>
<td>3.47</td>
<td>20,000</td>
<td>NOAA18</td>
</tr>
<tr>
<td>3/19</td>
<td>10:29:30-10:30:30</td>
<td>10.90</td>
<td>3.33</td>
<td>3.28</td>
<td>3.37</td>
<td>100,000</td>
<td>MetOp2</td>
</tr>
<tr>
<td>3/20</td>
<td>04:16:00-04:17:00</td>
<td>10.30</td>
<td>3.29</td>
<td>3.24</td>
<td>3.33</td>
<td>40,000</td>
<td>MetOp1</td>
</tr>
<tr>
<td>3/20</td>
<td>04:25:30-04:26:30</td>
<td>3.50</td>
<td>3.17</td>
<td>3.04</td>
<td>3.30</td>
<td>1,000</td>
<td>NOAA19</td>
</tr>
<tr>
<td>3/20</td>
<td>14:37:00-14:38:00</td>
<td>2.20</td>
<td>3.15</td>
<td>3.14</td>
<td>3.16</td>
<td>1,000</td>
<td>NOAA19</td>
</tr>
<tr>
<td>3/20</td>
<td>15:07:30-15:08:00</td>
<td>16.30</td>
<td>3.12</td>
<td>3.08</td>
<td>3.15</td>
<td>10,000</td>
<td>NOAA18</td>
</tr>
<tr>
<td>3/20</td>
<td>19:42:30-19:43:30</td>
<td>3.00</td>
<td>3.07</td>
<td>3.04</td>
<td>3.09</td>
<td>2,000</td>
<td>NOAA19</td>
</tr>
</tbody>
</table>
6. NOAA/MetOp Observations

For Cases 1 and 2 we carried out a survey of ring current proton precipitation events as measured by the Medium Energy Proton and Electron Detector (MEPED) instrument carried by a set of low-altitude near-polar orbiting POES (NOAA and MetOp) spacecraft. MEPED (Evans & Greer, 2000) measures ion fluxes in three energy bands (30–80 keV, 80–250 keV, and 250–800 keV; Engebretson et al., 2015). From six spacecraft (MetOp 1 and 2 and NOAA 15, 16, 18, and 19) we identified a total of approximately 300 events for the two storm cases. From these, we selected 40 “best” events. The characteristic POES ion precipitation event is described as a latitudinally narrow, strong peak in ion counts in the 0° detector in the 30–80 keV channel. Accumulated experience dating back at least to the work of Lundblad and Søraas (1978) and Søraas et al. (1980, 1999) strongly suggests that such a peaked precipitation flux represents ions that are precipitating as a result of EMIC wave scattering. The “best” events that we selected were the most intense and clearest, including a few of moderate intensity. Of the 300 initially selected POES events, many were less intense and others were either too complex or marginal to be identified with confidence as being related to EMIC waves. A high count rate was regarded as 10,000 (cm² s sr keV)⁻¹ or over and a moderate count rate as 1,000–10,000 (cm² s sr keV)⁻¹. The count rate is a possible proxy for EMIC wave intensity.

A selection of best proton precipitation events for Case 1 is given in Table 1, where we have selected events near \( \text{L} = 4 \) during the storm recovery phase, 18–20 March 2013. This is to coincide with the approximate location and times of the maximum proton spectral intensity as identified from RBSPICE-B and described in section 3. Regarding the notation in Table 1, the proton precipitation flux extends spatially from \( \text{Min} \text{L} \) to \( \text{Max} \text{L} \) and has a maximum intensity at \( \text{Peak} \text{L} \); amplitude refers to precipitation flux maximum intensity (cm² s sr keV)⁻¹. Similarly in Table 2 we give a selection of best precipitation events for Case 2. Here we choose events near \( \text{L} = 3 \) during the storm recovery period, 18–20 March 2015. The “amplitude” in Tables 1 and 2 was estimated from the time series plots of MEPED data created using the CDAWEB interactive plot facility.

In Figure 9 we illustrate three proton precipitation events selected from Table 1, and in Figure 10, we show three events selected from Table 2. In both figures the black trace (derived from the 0° pitch angle detector) shows the precipitating protons and the red trace (from the 90° detector) shows the trapped protons. Generally, regarding measurements from the POES MEPED instrument, a peak in the 30–80 keV channel is often (but not always) accompanied by a similar weaker peak in the 80–250 keV channel. Peaks in both the 30–80 keV and 80–250 keV channels can be observed in all the events shown in Figures 9 and 10. Further, for each MetOp 1 event in Figure 9, peaks in both the 30–80 keV and 80–250 keV channels are accompanied as well by weaker peaks in the 250–800 keV channel.

EMIC wave-induced proton precipitation events that we identify from POES during the storm recovery periods of Cases 1 and 2 are also indicated by vertical lines in the bottom panels of Figures 11 and 12, respectively; the colors of the vertical lines in the bottom panel...
indicate the approximate \(L\)-shells at which ring current proton precipitation was observed. Evidently, the POES results we have reported indicate significant EMIC wave activity during the recovery phase of both storms.

We note the recent statistical studies of EMIC waves using Van Allen Probes data by Saikin et al. (2015), Wang et al. (2015), and Zhang et al. (2016). Accordingly, in the following section, we consider only scattering by \(\text{H}^+\)-band and \(\text{He}^+\)-band waves and omit consideration of the less frequently occurring \(\text{O}^+\)-band waves.

7. Comparison of Observed Spectra With Limiting Solutions

In Figure 11 we compare observed proton spectra generated during the Case 1 magnetic storm period against limiting energy spectra. We calculate the limiting spectra by solving the integral equation (4) using a numerical scheme given by Press et al. (2007). We plot the observed spectra at \(L \approx 4\), measured during inbound passes in the left panels and outbound passes in the right panels, at selected times during the storm recovery period, 18–20 March 2013. The observed spectra are shown as solid curves, color coded according to UT and orbit number. In the bottom panel of Figure 11, which shows the time profile of the \(\text{Dst}\) index, the colors of the vertical lines indicate the \(L\)-shells of EMIC wave activity during the recovery period, as deduced from POES observations of ring current proton precipitation. In the top panels of Figure 11 we show the theoretical limiting spectra as dashed curves for the case of a hydrogen plasma \((\eta_1 = 1, \eta_2 = 0, \eta_3 = 0)\) for two different values of the parameter \(a = \frac{|\Omega_e|^2}{\omega_{pe}^2}\); namely, \(a = 0.01\) and \(a = 0.1\). These \(a\)-values correspond respectively to the approximate background electron number density values \(N_0 = 231\ \text{cm}^{-3}\) and \(N_0 = 23.1\ \text{cm}^{-3}\), at \(L = 4\), assuming a dipole magnetic field. The middle panels of Figure 11 similarly show limiting spectra as dashed curves for the case of a hydrogen-helium plasma \((\eta_1 = 0.75, \eta_2 = 0.25, \eta_3 = 0)\) for the aforementioned values of the parameter \(a\). In the top and middle panels we also show the theoretical asymptotic solution given by equation (8) as a solid colored line. In all cases shown in Figure 11 we have set the pitch angle index \(s = 0.2\).

Regarding the value of the wave gain \(G\), which we also require in the solution of equation (4), we set \(G = 1.5\), which again is a commonly assumed value in the calculation of self-limiting fluxes, for example, Kennel and Petschek (1966), Schulz and Davidson (1988), and Mauk and Fox (2010).

Within the limits of the uncertainty of the (appropriately averaged) background values of the electron number density \(N_0\), Figure 11 indicates generally that the observed proton spectra are remarkably close to the theoretical limiting spectra. For the case of a hydrogen-helium background plasma, there is relatively close agreement between the observed spectra and the limiting spectra over the energy range from 50 keV to 200 keV. For the case of a hydrogen background plasma, close agreement between the observed and limiting spectra over the range from 50 keV to 200 keV is limited to the lower density \((a = 0.1)\).
case, while for the higher density ($a = 0.01$) case, such close agreement occurs over the more-limited energy range, 150 keV to 200 keV, approximately.

In Figure 12 we show a comparison of observed energetic proton spectra (solid curves) with theoretical limiting spectra (dashed curves) for the Case 2 storm period. The format of Figure 12 is similar to that of Figure 11, except that for Figure 12, we set $L \sim 3$ and select observed spectra during the storm recovery period 18–20 March 2015. In this case the chosen $a$-values, $a = 0.01$ and $a = 0.1$ correspond respectively to $N_0 = 1298$ cm$^{-3}$ and $N_0 = 129.8$ cm$^{-3}$ at $L = 3$ for a dipole field. The colors of the vertical lines in the bottom panel indicate the $L$-shells at which ring current proton precipitation was observed.

Figure 11. Comparison of RBSPICE-B proton spectra (solid curves) for Case 1 ($L \sim 4$) with theoretical limiting spectra (dashed curves). The solid colored line in each of the upper four panels represents the theoretical asymptotic solution (8). The vertical colored lines in the bottom panel represent EMIC wave activity as deduced from proton precipitation events observed by MEPED. The colors of the vertical lines indicate the $L$-shells at which ring current proton precipitation was observed.

In Figure 12 we show a comparison of observed energetic proton spectra (solid curves) with theoretical limiting spectra (dashed curves) for the Case 2 storm period. The format of Figure 12 is similar to that of Figure 11, except that for Figure 12, we set $L \sim 3$ and select observed spectra during the storm recovery period 18–20 March 2015. In this case the chosen $a$-values, $a = 0.01$ and $a = 0.1$ correspond respectively to $N_0 = 1298$ cm$^{-3}$ and $N_0 = 129.8$ cm$^{-3}$ at $L = 3$ for a dipole field. The colors of the vertical lines in the bottom panel indicate the $L$-shells at which ring current proton precipitation was observed.

For Case 2 we similarly assume that $s = 0.2$ and $G = 1.5$. We also consider the cases of a hydrogen plasma ($\eta_1 = 1$, $\eta_2 = 0$, $\eta_3 = 0$) in the top panels and a hydrogen-helium plasma ($\eta_1 = 0.75$, $\eta_2 = 0.25$, $\eta_3 = 0$) in the middle panels. We see from Figure 12 that agreement between the observed and limiting spectra is quite good over the proton energy range, 100 keV to 300 keV, for the case of a hydrogen-helium plasma. Such reasonable agreement occurs for the case of a hydrogen plasma over the energy range 100 keV to 300 keV only for the lower density ($a = 0.1$) case. Over the lower energy range, 50 keV to 100 keV, for Case 2, the observed spectra fall somewhat below the theoretical limiting spectra, as shown in each of the top and middle panels of Figure 12.
The results reported in Figures 11 and 12 provide evidence that proton fluxes are limited by the action of EMIC waves during the recovery periods of both storms over limited energy ranges.

8. Summary and Conclusions

1. We have reported measurements of energetic proton spectra from the RBSPICE-B instrument on the Van Allen Probes, in the region $3 \leq L \leq 6$, during two magnetic storm periods, namely, a strong storm, Case 1, 17–20 March 2013, and a severe storm, Case 2, 17–20 March 2015.

2. Typically, we find that for Case 1, the most intense proton spectra occur at energies, 50 keV–200 keV, in the region $4 \leq L < 5$ during the storm recovery period 18–20 March 2013, while for Case 2, the most intense spectra occur for $3 \leq L < 4$ at energies 50 keV–300 keV during the recovery period 18–20 March 2015.

3. From a survey of ring current proton precipitation events measured by the MEPED instrument on six POES (NOAA and MetOp) spacecraft, we deduce that EMIC wave activity was significant during the recovery phase of both storms, in particular at $L \sim 4$ for Case 1 and at $L \sim 3$ for Case 2.

4. We present a theory that determines the maximum (limiting) ring current proton spectrum resulting from EMIC wave-proton interactions. The limiting spectrum satisfies an integral equation that we solve numerically.

The results reported in Figures 11 and 12 provide evidence that proton fluxes are limited by the action of EMIC waves during the recovery periods of both storms over limited energy ranges.

**Figure 12.** Comparison of RBSPICE-B proton spectra (solid curves) for Case 2 ($L \sim 3$) with theoretical limiting spectra (dashed curves). The solid colored line in each of the upper four panels represents the theoretical asymptotic solution (8). The vertical colored lines in the bottom panel represent EMIC wave activity as deduced from proton precipitation events observed by MEPED. The colors of the vertical lines indicate the $L$-shells at which ring current proton precipitation was observed.
Acknowledgments
This project is supported by a Discovery Grant of the Natural Sciences and Engineering Research Council of Canada to D.S. Proton data used in Figures 2–8 and 11 and 12 are available from RBSPICE via the website http://spice.gsfc.nasa.gov. The CDAWeb data plots from the NOAA and MetOp satellites in Figures 9 and 10 were created using the interactive plot facility provided by the Coordinated Data Analysis Web (CDAWeb) at http://cdaweb.gsfc.nasa.gov. We thank David Braun of Augsburg College for assistance in identifying precipitating proton events in the POES data. Work at Augsburg College was supported by U.S. National Science Foundation grants AGS-1264146 and PLR-1341493. K.O. is supported by the Research Council of Norway under contract 223252. K.O. is also grateful for being selected as the 2017 Fulbright Arctic Chair, and his sabbatical at Virginia Tech is sponsored by the U.S. National Fulbright Foundation for Educational Exchange.

References


