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Key Points:
- The BV technique is benchmarked with respect to other single-spacecraft methods
- It is less sensitive to noise than most of the other methods on simulated data
- A statistical study is made on 149 Cluster magnetopause crossings

Supporting Information:
- Text S1
- Table S1

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Magnetopause orientation: Comparison between generic residue analysis and BV method

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Abstract Determining the direction normal to the magnetopause layer is a key step for any study of this boundary. Various techniques have been developed for this purpose. We focus here on generic residue analysis (GRA) methods, which are based on conservation laws, and the new iterative BV method, where B represents the magnetic field and V refers to the ion velocity. This method relies on a fit of the magnetic field hodogram against a modeled geometrical shape and on the way this hodogram is described in time. These two methods have different underlying model assumptions and validity ranges. We compare here magnetopause normals predicted by BV and GRA methods to better understand the sensitivity of each method on small departures from its own physical hypotheses. This comparison is carried out first on artificial data with magnetopause-like noise. Then a statistical study is carried out using a list of 149 flank and dayside magnetopause crossings from Cluster data where the BV method is applicable, i.e., where the magnetopause involves a single-layer current sheet, with a crudely C-shaped magnetic hodogram. These two comparisons validate the quality of the BV method for all these cases where it is applicable. The method provides quite reliable normal directions in all these cases, even when the boundary is moving with a varying velocity, which distorts noticeably the results of most of the other methods.

1. Introduction

The Earth's magnetopause is the boundary between the cold and dense plasma of the magnetosheath, i.e., the shocked solar wind, and the magnetosphere, dominated by the Earth's magnetic field, where the plasma is 1 order of magnitude hotter and tenuous. Understanding the structure of this kind of boundary between different plasmas and magnetic fields is important for our understanding of very general physical phenomena, such as magnetic reconnection and surface wave instabilities (e.g., tearing modes and Kelvin-Helmholtz). Nevertheless, the observational study of the magnetopause from spacecraft data is made difficult by the fact that the boundary is unsteady, due to solar wind variations and/or to various kinds of waves (coming from the magnetosheath or created locally by surface instabilities). Several methods have been developed for the purpose of finding at least the normal direction to the boundary. We will here focus on those that use only the data of one spacecraft: the "single-spacecraft" methods. Some of these methods can provide in addition various physical characteristics of the boundary, as the normal magnetic field B_N, velocity u_N, and even a coordinate along this normal direction.

Most methods providing a normal direction are based on conservation laws. Assuming planarity and stationarity, the normal is then determined as the direction where these laws are best satisfied. For example, the well-known and often used MVAB (minimum variance analysis on the magnetic field) method uses the fact that $\nabla \cdot \mathbf{B} = 0$, which gives $B_N =$ constant in the 1-D case. A lot of such laws can be derived, forming a class of methods that are called "residue methods" in *Sonnerup et al.* [2006] (see the appendix for a short description of the different generic residue analysis (GRA) methods). The composite (COM) method, described in the same paper, combines these various conservation law methods, weighting each method by the ratio of intermediate and minimum variance eigenvalues. Higher eigenvalue ratio translates to a higher weight. These methods have been well benchmarked, and some have been compared with multispacecraft methods based on timing in *Haaland et al.* [2004]. In contrast, the BV technique [*Dorville et al.*, 2014a] has not yet been benchmarked in the same way. It allows for the determination of the normal of any "C-shaped" magnetopause crossing as first introduced by *Berchem and Russell* [1982] and later used by *Panov et al.* [2011]). We consider any magnetopause crossing with a convex hodogram in the *LM* plane at the crossing scale (i.e., independently of the shorter-scale variations due to noise and turbulence) as a C-shaped crossing. This categorization is performed qualitatively by visual inspection. The BV method provides in addition a coordinate along the normal direction, $y = \int_{t_0}^{t} V_N(t)\mathrm{d}t$. The description of the way the model hodogram is described in time depends on this coordinate, so the normal direction and normal coordinate are determined together and consistently. A small error on the normal direction would generally imply a description of the elliptical hodogram very far from linear. This property is precisely why the BV method is expected to be robust. The resulting normal coordinate can be used to study the spatial profiles of various physical quantities inside the boundary. Nevertheless, it cannot be benchmarked with respect to the GRA methods since these methods do not provide comparable outcome. It could be useful, in future works, to compare it to the "transition parameter" defined by *Lockwood and Hapgood* [1997], with the difference that the latter is independent of a normal determination and is limited by the time resolution of the electron experiment.

The BV technique has some underlying assumptions that are different from those of the generic residue analysis (GRA) methods. It is based on a combination of the velocity and magnetic field data: it consists of fitting the magnetic field data with an elliptical model with an *LM* plane polar angle that varies linearly as the satellite moves along the normal direction at the normal ion velocity. At the moment, the BV technique has only been validated against simulations and for a few case studies [see *Dorville et al.*, 2014a, 2014b]. Its differences with other single-spacecraft methods have been theoretically presented by *Dorville et al.* [2014a], but it has not been statistically validated using observations.

The purpose of this paper is to benchmark the BV method statistically and to compare it with the GRA methods in finding the normal direction of the magnetopause. This is carried out in two steps. First, in section 2 we present a study on simulated data to understand the sensitivity of different methods to noise and to small departures from their underlying hypotheses. The type of applied noise will be described in section 2. Then, the rest of the paper focuses on a statistical study based on a list of 149 C-shaped magnetopause crossings. In section 3, we present the methodology of the study and its main results. Then we conclude in section 3.2 the validation of the BV technique and its comparison to GRA methods. This comparison will focus on the normal direction determination, and not on the coordinate provided by the BV method, which cannot be benchmarked with respect to GRA.

2. Comparison on Artificial Data

In this section, we present some tests carried out on artificial data to benchmark the BV method and compare its sensitivity to noise with those of MVAB,MVAB0 (a constrained minimum variance analysis in which the assumption that the average normal magnetic field vanishes is enforced), MVAE (that uses the direction of maximum variance of the electric field), and MMR (minimum massflow residue; conservation of mass along the normal direction) methods. We first summarize the principles of these methods then explain the numerical experiment and finally comment on the results and the conclusions we draw from it.

2.1. The Methods Under Comparison

The BV method [*Dorville et al.*, 2014a] addresses in the same process two of the main issues concerning the analysis of magnetopause-like discontinuities using spacecraft data. Its purpose is to determine (i) the direction normal to the boundary and (ii) a spatial coordinate along it. The second point should allow to correctly distinguish spatial variations from temporal ones, even with single-spacecraft data and even for small-scale structures inaccessible to the four-spacecraft techniques of Cluster [see *Dunlop et al.*, 2002]. The method is presently designed to work only on the so-called C-shaped magnetopause crossings [*Berchem and Russell*, 1982], as illustrated by *Panov et al.* [2011] in using Cluster magnetic field hodograms. Physically speaking, the tangential magnetic field varies with a C-shaped hodogram when the magnetopause boundary has a simple structure, with a parallel current (providing the rotation) flowing in the same direction all along the crossing. It can also involve a perpendicular current (related to compression), but this component has also a simple structure and occurs approximately at the same place and on the same scale as the parallel one. From the observations, the C-shaped hodograms are selected by visual inspection, based on the criterion that they are convex at the large scale of the crossing. Other types of crossings may involve multiple sublayers where the compressional and rotational components can be separated [e.g., *Dorville et al.*, 2014b] or else being consistent with adjacent current sheets that display different orientations of the current sheets, e.g., S-shaped crossings [*Panov et al.*, 2011]. These configurations are likely to be associated with strong perturbations of the boundary, due, for instance, to reconnection. The BV method is not able, at the moment, to analyze such kind of crossings. For the C-shaped crossings analysis, the determination of the normal by the BV method is done by combining the analysis of magnetic field data and ion velocity. First, it is assumed that the magnetic field of a hodogram can be fitted by a simple elliptical shape in a plane perpendicular to the unknown normal direction. Second, it is assumed that the angle of the in-plane elliptical model can be described linearly in space and that the temporal description can simply be derived from the normal component V_N of the measured ion velocity. Combining these two pieces of information provides a robust determination of the normal since an erroneous determination would imply two adverse conditions at the same time: (1) the normal magnetic field is hardly distinguishable from one of the tangential components (as for MVAB) and (2) the normal velocity is hardly distinguishable from the tangential velocity (value and variations). The method demands a careful implementation, which has been fully described by *Dorville et al.* [2014a]. The main principles are briefly summarized in Appendix B. The consequences of its hypotheses, such as the linear relation chosen to link position and angle, will be evaluated in the present benchmark analysis.

In this section the BV method is compared to some of the residue methods presented in *Sonnerup and Scheible* [1998]. The GRA methods employed here are summarized in Appendix A. The first one is MVAB, for “minimum variance analysis on the magnetic field.” Its principle is to compute and diagonalize the variance-covariance matrix of the magnetic field, in order to determine which direction corresponds to the most constant component of the magnetic field and identify it as the normal direction. MVAB provides three characteristic directions L, M, and N corresponding, respectively, to the maximum, intermediate, and minimum variance of the magnetic field. The subscripts L, M, and N will denote the vector components in these three directions throughout the rest of this paper. The weakness of this method is the fact that the field can sometimes also be quite constant along the intermediate variance direction M (quasi-coplanar cases). Although the eigenvalues corresponding to the M and N directions are both small in this case, their ratio is not necessarily small. This ratio mostly depends on the superposed fluctuations (noise/turbulence) and particularly on their isotropy. The variance related with these fluctuations is often larger than the variance due to the jumps that characterize the discontinuity itself. In these rather common conditions, it is worth remembering that a large ratio of the M and N eigenvalues is actually not a sufficient condition to ensure a valid result of the MVAB method. To bypass this problem, one can perform a constrained analysis in which the condition $B_N = 0$ is added (see also Appendix A). This is the MVAB0 method. But this strong hypothesis is not justified when the magnetopause is reconnected (identified, for instance, as a rotational discontinuity), and it precludes in any case to determine the B_N component. We will also compare the BV method to the maximization of the variance of $v \times B$ (MVAE) and to the mass flux conservation methods (MMR), which are both sensitive to the variations of the normal boundary velocity, since these methods suppose that it is constant during the crossing.

2.2. Numerical Experiment

We will first examine simulated data, taking advantage of the fact that in this case, the exact normal is perfectly known. One can thus study the deviations from this exact normal by the different methods. This also enables us to investigate the effect of small departures from the validity conditions of each method.

To reach this goal, we build a set of “artificial” magnetic field, ion velocity, and density measurements, providing C-shaped magnetic hodograms with Alfvénic speed and constant density inside the boundary. The artificial data set satisfies $\nabla \cdot \mathbf{B} = 0$ and the conservation of mass flux. We will investigate a rotational case (when the normal field is nonzero) and the tangential limit of these discontinuities, i.e., take the limit $B_N = 0$. Then, we perform three kinds of transformations on these artificial profiles: (1) we turn the vectors into a different frame, corresponding to the fact that the LMN frame is not known a priori in the experimental results; (2) we add an acceleration along the normal direction, which is the simplest way of taking into account a nontrivial motion of the boundary with respect to the spacecraft; and (3) we add random noise. We simulate a large-scale “magnetopause-like” noise as follows: we give random values at a few points along the crossing and then use polynomial interpolation between these points (a simple Gaussian noise would not resemble the fluctuations observed at the magnetopause; furthermore, we have verified that it would not disturb BV at all). Artificial data built in this way are shown in Figures 1 and 4. The noise level is about 5% in both cases.

Working on this artificial data set, we compute normal directions provided by different methods and repeat the experiment for 10 different sets of random noises. We can analyze the distribution of results from each method and obtain the mean departures from the predefined, actual, normal direction and the dispersion around this mean value. We can also compare results (mean values and variances) from one method to another. The results are presented on a polar plot, built like in *Haaland et al.* [2004]. The center of this polar plot

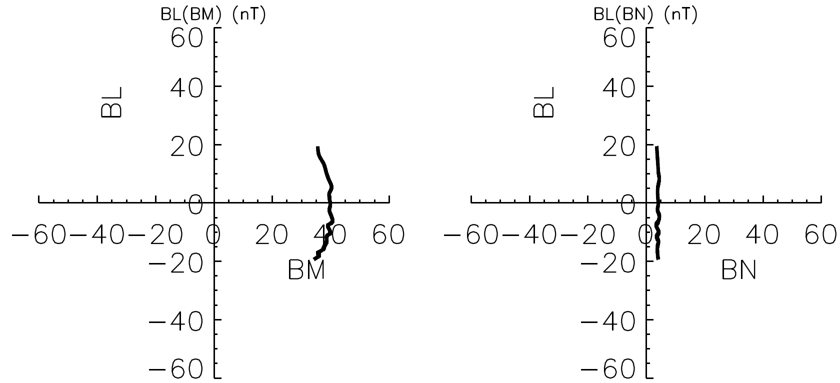

Figure 1. Typical hodogram of the magnetic field for a simulated crossing of a rotational discontinuity with noise. Shown are (left) $B_L(B_M)$ and (right) $B_L(B_N)$, in nT. During the same interval, $\mathbf{V}_T$ is proportional to $\mathbf{B}_T$, the density N is constant, and V_N is a linear function of time (representing the combination of the flow and the velocity of the boundary).

corresponds to the known true normal direction. The distance to the center indicates the angle between the vector found and this reference (labels on the axes are in radians), and the polar angle indicates its direction with respect to two axes of reference (here M and L). On this kind of plots, one can therefore see at the same time how far a normal direction is from the reference and in which direction on the L-M plane this deviation is.

2.3. Results and Discussion

We present here results of two runs, one with a rotational discontinuity (with $B_N = 4nT$ and other characteristics of this kind of discontinuity, like Alfvénic flow) and another with a tangential discontinuity ($B_N = 0$). The rotational case is varying on a circle. The tangential case is more arbitrary. It is chosen here to have a small variation of the B_M component and so a week curvature of the hodogram. In such cases, the normal determination is more difficult for most methods, making the comparison more enlightening. In each case, we keep the same large-scale structure for the magnetopause and repeat the experiment 10 times adding different random noise on the measured magnetic field. The boundary is accelerating from 30 km/s to 100 km/s during the crossing, and the noise amplitude is 5%. A typical hodogram of the magnetic field is shown in Figure 1. One can see the large-scale variation of the field and the noise superposed to it.

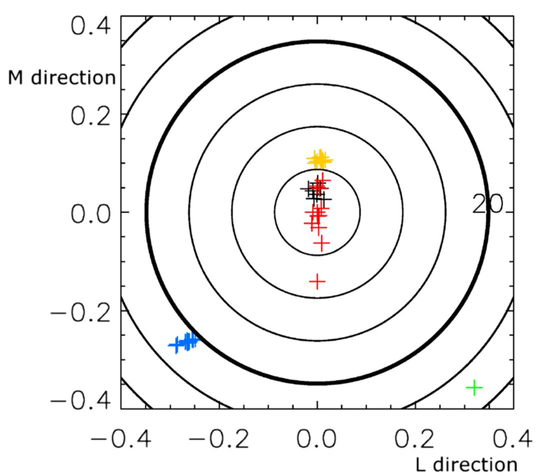

Figure 2. Position of the different normal directions found for a rotational discontinuity for different methods: BV (black), MVAB (red), MVAB0 (yellow), MVAE (blue), and MMR (green). Results are presented in a polar plot as described in the text. The center of the plot corresponds to the model normal direction (without noise). The distance to the center quantifies the angle between the normal found and the model normal direction in radians. Circles are drawn every 5°. A bold circle is drawn at 20°. The abscissa axis corresponds to the L direction and the ordinate axis to the M direction of the model.

Figure 2 presents the results of the normal determinations as a polar plot. The first conclusion is that the mass flux method gives a very bad result here (around 30° error), which is quite understandable since the velocity of the boundary is varying in a nonnegligible way. One can see that the method based on the variance of $\mathbf{v} \times \mathbf{B}$ is quite far from the center (again certainly because of the accelerating boundary) and that there is a nonnegligible dispersion of the MVAB points (with a clear anisotropy along the M direction), although the magnetic field chosen presents a reasonable curvature. The MVAB0 method appears less disturbed by the noise than the other methods, but it is in average 8° off, because of the nonnull B_N. The BV method shows a slightly larger dispersion but with an average value quite close to the exact result. Overall, for such cases where the normal magnetic field is not zero, BV seems to be a better and more stable choice than the other methods, at least in such cases where

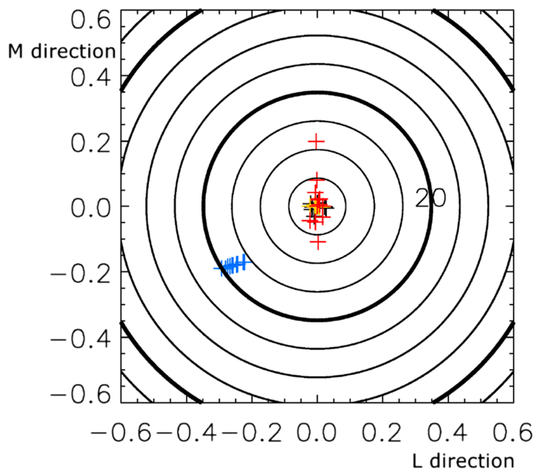

Figure 3. Position of the different normal directions found for a tangential discontinuity. Same format as Figure 2.

the necessity of distinguishing (B_N, V_N) and (B_M, V_M) is fulfilled. MVAB is then more disturbed by the noise, MVAB0 by the 4 nT normal field, and other methods by variations in the boundary velocity. However, on one occasion, BV did not succeed to converge on a consistent result, meaning that the quality of the fit must always be checked when using BV: the coefficients of correlation between the fit and the data provide the needed indication of the good convergence of the process.

For the tangential discontinuity example, the conclusions are almost similar (see Figure 3 where the results of the normal determination are presented). In this example, in addition to the null normal magnetic field, we have also chosen a more coplanar field (Figure 4 shows the hodogram of the field.). MVAB is then less stable due to the difficulty to distinguish the *M* and *N* directions (with again a clear anisotropy of the errors along the *M* direction), while MVAB0 becomes the best method, which could be expected since B_N is strictly null, which exactly corresponds to its hypothesis. BV still provides, however, a quite good normal, with relatively small dispersion. It also remains the only method that provides a normal coordinate.

For this study with artificial data, we had the possibility to choose the number of sampling points. We choose here a large number (about 1000 points in the gradient) so that the error ellipse resulting from *Sonnerup and Scheible's* [1998] formula has a size of less than 1° for the major axis. This formula actually comes from a linear estimation of the error due to sampling, in the presence of a Gaussian noise of small amplitude. We emphasize that this formula can be a strong underestimation of the angular error, even in the absence of other sources of systematic error, whenever the two smallest eigenvalues, *M* and *N*, appear to be dominated by the noise and not by the signal.

Having checked the quality of the BV method with respect to other methods on simulated data, we will now present a statistical study on real magnetopause crossings by the Cluster mission, in order to benchmark completely the BV method and compare it to the GRA methods.

3. Statistical Study on Real Magnetopause Crossings by Cluster

3.1. Goals and Methodology

The primary goal of this statistical comparison between BV and GRA methods is to benchmark the BV method. The hypotheses of the BV method have already been discussed and some comparisons made in *Dorville et al.* [2014a], but no exhaustive study has been carried out yet on a sufficiently complete list of magnetopause

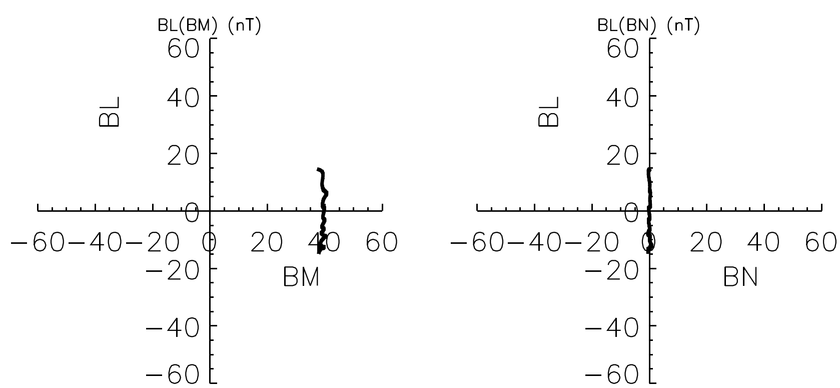

Figure 4. Typical hodogram of the magnetic field for a simulated crossing of a tangential discontinuity. Same format as Figure 1. During the same interval, V_T is proportional to B_T, N is constant, and V_N is a linear function of time.

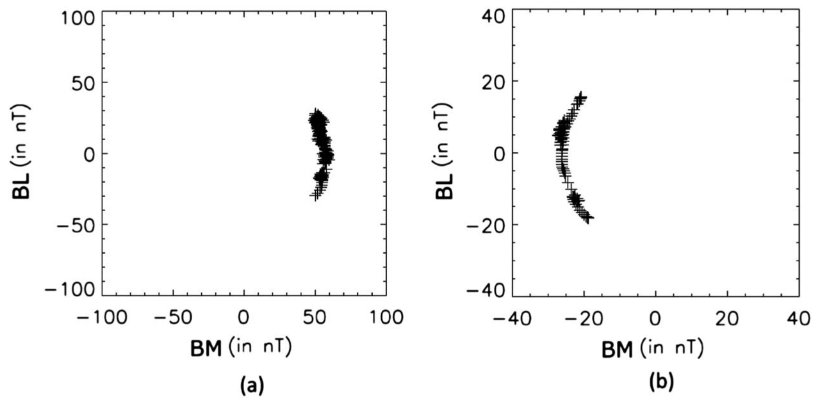

Figure 5. Hodogram of the magnetic field for two crossings from the 149 magnetopause crossings list, on (a) 28 June 2003 around 08:20 by C3 and (b) 29 May 2001 around 03:28 by C3. Both are considered as C shaped with our definition, Figure 5a being only slightly curved and Figure 5b looking really like a "C."

crossings. Another goal is to understand the similarities and differences with the other single-spacecraft methods and how these other methods compare with each other.

This study has been done on a set of 149 magnetopause crossings that we provide as supporting information. We first established a list where all the magnetopause encounters were selected by visual inspection. Then, further down selection was done to ensure that all the crossings under study could be analyzed by GRA. This implies having complete plasma and magnetic field data sets around each crossing time and a sufficiently good de Hoffman-Teller frame. For more details about the selection process, we refer to *Haaland and Gjerloev* [2013] for the list of flank magnetopause crossings and *Anekallu et al.* [2013] for the dayside magnetopause crossings.

Then, a last selection step is necessary to be able to use the BV method for which conditions are more restrictive. These crossings have to be convex at large scales, considering the magnetic field measured with the Fluxgate magnetometer (FGM) instrument (see *Balogh et al.* [1997] for details), and sufficiently long to get at least four to five plasma data points inside the boundary from the Cluster Ion Spectrometry (CIS) instrument (see *Rème et al.* [1997] for details), at a date and on a spacecraft where this instrument is able to provide velocity data (C1 or C3). A couple of typical magnetic field hodograms from the list are presented in Figure 5. The duration of the crossing is actually quite a restrictive condition on Cluster since the resolution of HIA (Hot Ion Analyzer) instrument is about 4 s. It should be easier to ensure with the MMS mission, giving access to a larger number of cases. We also tried to get comparable numbers of flank and dayside magnetopause crossings, and a list with a good coverage during the mission time and over a season, in order to have statistics as representative as possible for the cases accessible with the Cluster mission.

The list of 149 magnetopause crossings that we used is a result of the above three-step selection process, and it therefore contains only around 10% of the cases that can be analyzed with both BV and GRA methods. Out of these crossings, 77 are from flanks and 72 from dayside. The time span is going from 16 February 2001 to 14 July 2009. For each crossing, MVAB, MVAB0, MMR, MFR, MVAV, and the COM method with or without the $B_N = 0$ constraint [*Sonnerup and Scheible*, 1998] have been examined and compared to the BV method. The selected time intervals were identical for all methods. However, it is worth reminding that for its functioning, the BV method automatically selects a shorter interval inside this one in order to distinguish the current sheet proper and the two adjacent regions.

3.2. Results and Discussion

In this section, we present statistical results of normal directions obtained by BV method and different other methods. We present a comparison of magnetopause normal directions obtained from all these methods. The results are summarized in Figure 6. Each panel shows results of one method (all of the GRA methods that are included here are "constrained," i.e., assume $B_N = 0$). In each panel, each point corresponds to one of the 149 magnetopause crossings, and it shows the normal direction found by the method. For comparing these normal directions to the BV ones, the panels are built as follows: for each crossing, the center of the polar plot corresponds to the BV normal, the x axis corresponds to the L direction, and the y axis to the M direction

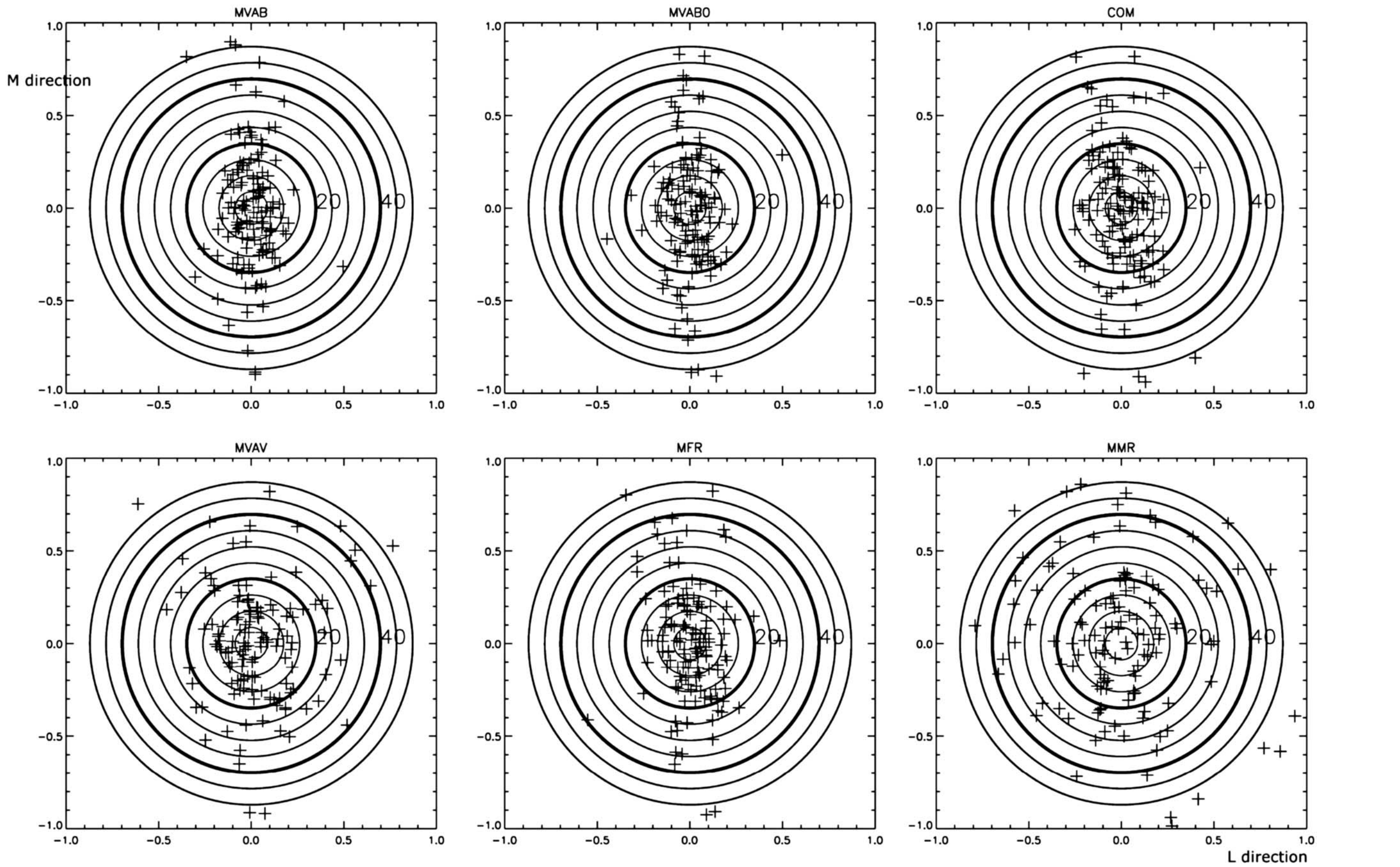

Figure 6. Position of the different normal directions found by MVAB, MVAB0, COM, MVAV, MFR, and MMR, for the 149 Cluster magnetopause crossings. Same format as Figure 2 except that the center of the plot corresponds here, for each case, to the normal direction found by BV and the abscissa and ordinate to the determined *L* and *M* directions.

(*L* and *M* directions are perpendicular to the *N* direction determined by BV and are crudely determined in this plane from MVAB and MVAB0).

The first point to be noted is that there is a spread of normal direction results around the BV normals. If we compute angles between BV normal and the different results, whatever the direction and mixing the methods altogether, we find a standard deviation about 8.2°. The corresponding deviation becomes 9.5° if the different methods are compared to the constrained COM instead of BV. This statistical result is not specific to the BV method (neither to the COM one): it is actually a good indicator of the accuracy that can be expected on average from the different methods. One can see in addition that other methods, such as MVAB, appear well concentrated around the center while others, as MMR, are much more dispersed. The big discrepancy of MMR compared to other methods could be explained by the difficulty to manage cold ions or composition effects.

The second striking feature appearing in Figure 6 is the strong anisotropy of the errors for all the methods except MVAV and MMR: this error is much larger in the *M* direction than in the *L* direction. This can easily be understood if one realizes that the main difficulty for all the methods based on the magnetic field is to distinguish between the *M* and *N* directions. This anisotropy is quite in agreement with the preceding results on artificial data (see Figures 2 and 3).

Concerning this anisotropy of the errors, we would also like to emphasize here that the usual criterion of a large ratio (for example, more than 5 or 10) between the *M* and *N* eigenvalues is not sufficient to prevent from a bad determination of the *N* direction in the *M*-*N* plane by MVAB.

This point can be illustrated with the example of a shock layer. In this case, it is well known that the use of MVAB is not recommended because, in an ideal case, there is no jump in the tangential direction perpendicular to *L*. Even if a small noncoplanar component is theoretically known to be possibly present inside the layer (but not on both sides), the variances in the *M*-*N* plane are likely to be dominated, in this case, by the turbulence always

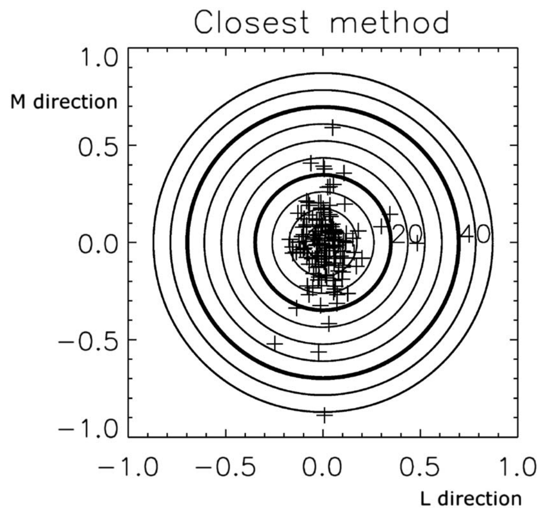

Figure 7. Position of normal direction closest to BV in the plane perpendicular to the normal direction found by BV for the 149 Cluster magnetopause crossings. Same format as Figure 6.

superposed to the jumps that characterize the discontinuity itself. A large ratio between the two eigenvalues in the *M-N* plane is therefore an indication of a strong anisotropy of the turbulence, and it may be the same if taking a data interval that contains the shock or not. In these conditions, the direction *N* found is therefore likely to be a characteristic of the turbulence rather than a characteristic of the discontinuity.

From the series of angles between the BV normals and the normal directions provided by the other methods, independently of the direction *L* or *M* of this deviation, a statistical study has been performed. An important result emerges from this study: when applicable, BV generally falls close to one of the other methods (this other method is not always the same). This point is illustrated in Figure 7. This figure is built in the same way as Figure 6, but each point comes here from the method that provides the normal closest to BV, instead of coming from a fixed method. The value of the angle between the BV method and the closest method is 8.7° in average, less than 10° for 71% of the cases and less than 25° for 97% of the cases. On the series of 149 crossings, only four gives a BV result far from all the other methods. These numbers are a priori sufficient to conclude that the present benchmark study validates the BV method.

One point deserves special attention: in the above study, the method that is most often the closest one with respect to BV is MVAB and not COM. This result is actually quite sensitive to the way COM is built. Up to now, the COM method used included the constrained methods (i.e., assuming $B_N = 0$) and not the unconstrained MVAB. It is important to know that when including MVAB in COM, 24 cases where MVAB was the closest method become cases where COM is the closest method. This COM method then becomes almost as frequent as the MVAB one (40 cases against 45). If one employs COM0, the version of COM that contains only constrained methods and not MVAB, it is important noticing that this method actually provides significantly different results from the complete COM one, even if its variance is not much different. The average angular difference $< \theta_{\mathrm{BV/COM}} - \theta_{\mathrm{BV/COM0}} >$ is 8.6°, i.e., it is not very different from the mean standard deviation of $\theta_{\mathrm{BV/other\ methods}}$. When using the GRA methods, one has therefore to take into account the fact that choosing COM or COM0 has an effect on the result of the calculation.

In Figure 8, the ordinate represents the number of cases, in percent, where the angle difference between the BV normal and the normal provided by a different method is smaller than the angle given in abscissa. This figure provides a summary of the above results under a different form. The big difference between the curve “minimum” and the methods studied separately shows that the results of the methods present a big dispersion for each case but that there is generally one or several methods that are much closer to BV. The fact that most of the methods are following quite the same curve shows that this closest method is not the same for each crossing. Finally, one can see that the methods based solely on plasma data, MMR and MVAV, have most often bad performances.

With these experimental data, we can compare the dispersion of the results with the statistical error ellipse due to data sampling [see *Sonnerup and Scheible*, 1998, equation (8.23)]. For MVAB and MVAB0, which can have a good time resolution (here 20 points/s) since they do not use ion data, this statistical error is between 2 and 4° for the major axis of the error ellipse. This statistical error is much smaller than the differences observed between MVAB and MVAB0 (and other methods) in most cases. One is therefore led to think that this equation, which takes only into account statistical noise and not any model breakdown, can lead to a strong underestimation of the angular error, even in the absence of any other source of systematic error in the data. This point was already clearly stated in the original paper of *Sonnerup and Scheible* [1998]. This underestimation of the angular error by the statistical calculation seems to occur whenever the directions corresponding to the two smallest eigenvalues, *M* and *N*, appear to be dominated by the small-scale structures (e.g., “noise”) and not

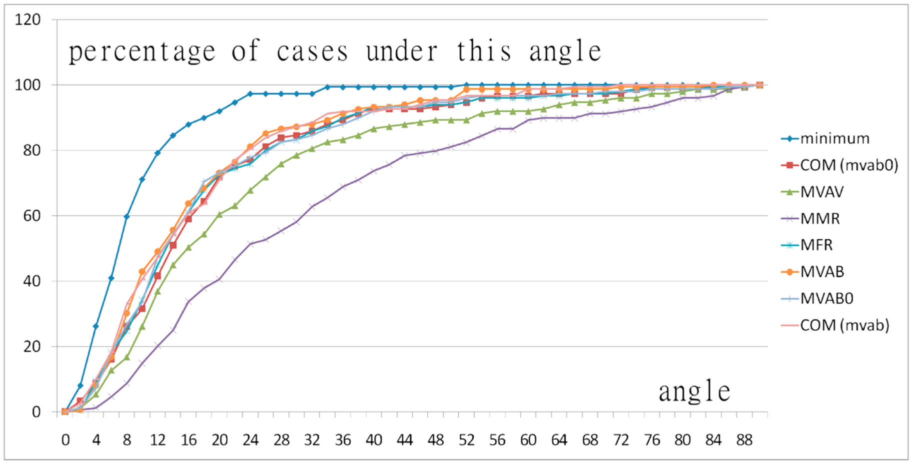

Figure 8. Percentage of cases when a given method is separated from BV by less than a certain angle in degrees. The comparison is made for each of the GRA methods studied (including constrained and unconstrained COM methods) and with the closest method for each case (curve min).

by the large-scale variation. This common situation is of course beyond the range of validity of the formula, since the formula derives from a linear calculation involving an uncorrelated noise: it therefore assumes that for all components, the noise has an amplitude much smaller than the signal and that it is a Gaussian one.

4. Conclusions

The purpose of this paper is to study the properties of the BV method concerning the determination of the magnetopause normal and to benchmark it against the other existing spacecraft methods. Two kinds of tests have been carried out, one numerical using artificial data and the other statistical using 149 magnetopause crossings observed by Cluster. As it could be expected, both studies show that the different single-spacecraft methods can give significantly different results. In the numerical study, where the exact result is known, the deviations have been shown to be sometimes very large, exceeding 30° in some occasions. The reasons of these deviations can then be easily explained by the small deviations, which were voluntarily introduced in the artificial data, from the underlying assumptions of the different methods. This preliminary study also allows understanding the differences observed in the statistical results with real data to a large extent.

Concerning the validation of the BV method, the numerical study verifies its good quality since it is shown that it is not very sensitive to the small-scale fluctuations and noise in the data, its dispersion range being low (typically less than 5°; see Figure 2). The conditions of applicability of BV are confirmed by this study: it gives valuable results whenever the magnetic variations are C shaped across the boundary, i.e., when the *M* and *N* directions can be distinguished *either* in the magnetic field *or* in the velocity field.

The statistical study also well demonstrates the usefulness of the BV method, at least for the list of crossings selected: in order to be analyzed by BV, the crossings have to support the above C-shaped hodogram condition and have a duration long enough to include a few points in the ion data (at least four to five plasma points inside the boundary). This study also allows a benchmark of the BV method: its normal is, in most of the cases investigated, close to the normal found by one of the GRA methods. BV happens not to converge in a few occasions, but it is generally not misleading: its result is then far from all the other methods (more than 30° for both the numerical experiment and the statistical one) so that it can easily be discarded.

Due to the low time resolution of Cluster plasma moments, the duration of the individual magnetopause crossings has restricted the selection. In the future, with the arrival of the MMS mission, this problem will be greatly reduced. We can be confident that the BV method will then be useful in order to study spatial profiles inside the layer and better understand its detailed structure.

Appendix A: Generic Residue Analysis

Generic residue analysis to determine the orientation and motion of a boundary [*Sonnerup et al.*, 2006] is based on residue analysis of conservation laws. This procedure incorporates the magnetohydrodynamic conservation laws for mass, energy, momentum, and entropy. A similar scheme can also be adapted for conservation laws derived from Maxwell's equations: magnetic flux conservation, conservation of magnetic poles, and conservation of electric charge.

A1. Basics of the Methods

Following the approach of *Sonnerup et al.* [2006] and employing the Einstein notation, a generic conservation law can be expressed as

$$\frac{\partial \eta_i}{\partial t} + \frac{\partial q_{ij}}{\partial x_j} = 0 \tag{A1}$$

where η_i is the density of the conserved quantity and q_{ij} is the corresponding transport tensor. If the conserved quantity is a scalar, e.g., mass ($\eta_i \equiv \eta = \rho$), the index i is simply dropped, and $q_{ij} \equiv q_j = \rho v_i$ is then a transport vector.

Assume now that the one-dimensional, time-invariant discontinuity moves with a constant speed u_N along the yet unknown normal $\vec{N}$. In this comoving frame, the time dependence disappears and equation (A1) can be written

$$-u_N \frac{\mathrm{d}\eta_i}{\mathrm{d}x'} + \frac{\mathrm{d}(n_j q_{ij})}{\mathrm{d}x'} = 0 \tag{A2}$$

where x' are the coordinates of the comoving system. Integrated across the discontinuity, this gives

$$-\eta_i u_N + n_j q_{ij} = C \tag{A3}$$

where C is an integration constant.

For real discontinuities, there will always be deviations from the ideal one-dimensional, time-invariant model above; equations (A1) and (A2) will therefore not be perfectly satisfied for any measurement across the boundary layer, but the best result can be obtained by minimizing the residue:

$$\begin{aligned} R &= \frac{1}{K}\sum_{k=1}^{k=K}\left|-\eta_i^{(k)}u_N + n_j q_{ij}^{(k)} C_i\right|^2 \\ &= \left\langle \left|-\eta_i^{(k)}u_N + n_j q_{ij}^{(k)} C_i\right|^2 \right\rangle \end{aligned} \tag{A4}$$

This expression can be solved for the optimal values of $\vec{C}*$ and $\vec{u}*$, where the latter is the optimal velocity of the discontinuity. The resulting matrix has the form $n_i Q_{ij} n_j$, where Q_{ij} is a symmetric matrix similar to covariance matrices known from minimum variance analysis. The eigenvectors of this matrix determine the orientation of the discontinuity; the eigenvector $\vec{x}_3$ corresponding to the smallest eigenvalue, λ_3, gives the normal of the discontinuity. Similarly, the eigenvalue ratio provides information on how well the eigenvectors are resolved.

A number of specific conservation laws can be formulated and treated according to the above scheme. For a complete description, we refer to *Sonnerup et al.* [2006]. The variants used for the comparison in this paper are as follows:

Minimum Variance Analysis of the Magnetic Field. MVAB, first applied by *Sonnerup and Cahill* [1967] for discontinuity analysis of magnetopause traversals, has become a standard method to determine the boundary normal of a discontinuity. The underlying physics is the conservation of magnetic solenoidality (expressed as $\nabla \cdot \vec{B} = 0$), which can be cast into the generic form of equation (A1). The plasma flow along this normal gives a rough idea about the velocity, at least in cases with no reconnection and thus no plasma flow across the magnetopause. A better estimate of the speed of a discontinuity is often obtained from *de Hoffmann-Teller analysis (HT)*, in which one tries to find a frame of reference where the electric field disappears, i.e., a frame comoving with the discontinuity. A detailed discussion about HT analysis and MVAB can also be found in *Sonnerup et al.* [2006] and *Khrabrov and Sonnerup* [1998b].

Maximum Variance of the Electric Field. MVAE is based on Faraday's law which requires that the two tangential components of the electric fields remain constant throughout the current layer. The normal component, however, typically undergoes a large change as a consequence of tangential flow and a change of magnetic field direction from one side of the layer to the other. The direction of maximum variance of the E field thus serves as an estimate of the boundary normal. In practice, the full 3-D electric field is rarely available, and the proxy $\vec{E} = -\vec{V} \times \vec{B}$ is often used.

Minimum Variance Analysis of Velocity. MVAV is in many ways similar to MVAB. The underlying assumption is that in the comoving frame, there is no divergence or variance in the plasma along the normal orientation. In our analysis, the plasma velocity is based on CIS-HIA ion moments.

Minimum Faraday Residue Analysis. MFR is based on conservation of magnetic flux across a boundary and also utilizes Faraday's law across the magnetopause current layer to find a moving frame and orientation such that the tangential component of the electric field is as constant as the data permit. As for MVAE, the electric field is often calculated from the plasma velocity via $\vec{E} = -\vec{V} \times \vec{B}$. MFR returns both a normal and a velocity of the discontinuity. An alternative approach is shown in *Khrabrov and Sonnerup* [1998a].

Minimum Massflow Residue Analysis. MMR is based on conservation of mass across the discontinuity and does not require any information about the magnetic field across the boundary. MMR can provide both orientation and velocity of a discontinuity. Since MMR relies on mass density, the presence of heavy ions can be a problem if the plasma measurements do not resolve composition. Likewise, the presence of cold ions with energies below the effective instrument energy threshold will lead to an underestimate of the density.

A2. Combining Information From More Methods or More Spacecraft

Results from two or more of the above residue methods can be combined to produce a single estimation of the orientation and velocity. This is done by adding a set of suitable weighted and normalized covariance matrices (Q matrices; see equation) and then calculate the eigenvalues and eigenvectors of the combined matrix. The weighting and normalization of the individual Q matrix is not unique, but it is often desirable to put more emphasis on results where the eigenvalues are well separated.

A composite matrix with weights, w_k for each method, can thus be expressed

$$Q_{\mathrm{COM}_{ij}} = \sum_{k=1}^{k=K} w^{(k)} Q_{ij}^{(k)} \tag{A5}$$

The composite normal of the discontinuity is then the eigenvector $\vec{x}_3$ corresponding to the smallest eigenvalue, λ_3.

Similarly, a composite velocity of the discontinuity can be obtained

$$\vec{U}^{*}{}_{\mathrm{COM}} = \sum_{k=1}^{k=K} w^{(k)} \vec{U}_k^{*} \tag{A6}$$

The composite method thus utilizes all available data from a discontinuity and may reveal properties not immediately seen if only one of the methods were used.

As a variant of the composite method, one may also add Q matrices from different spacecraft or combine data vectors from more spacecraft in the above methods. This may be useful in cases where the discontinuity is thin, and only a handful of measurements are available from each spacecraft.

A3. Constrain the Variance Analysis

If one has some a priori knowledge about the type of discontinuity studied, it is often desirable to constrain the analysis so that this knowledge is taken into account. For example, ideal tangential discontinuities have no normal magnetic field or flow across, i.e., $B_N = 0$ and $V_N = 0$. One can then modify the covariance matrix, Q_{ij}, so that this property is always satisfied. To ensure $B_N = 0$, one has to do the analysis so that the predicted normal, $\vec{N}$, is perpendicular to the direction of the average magnetic field $\vec{e} = <\vec{B}>/|<\vec{B}>|$. The most convenient way to impose such constraints is to perform a double projection on the covariance matrix, i.e., replace the original covariance matrix with $Q'_{ij} = P_{ik} Q P_{nj}$ where the projection matrix is given by

$$P_{ij} = \Delta_{ij} - e_i e_j \tag{A7}$$

The eigenvectors of the modified matrix, Q'_{ij}, now have a different meaning. Since we introduce a known quantity, the vector $\vec{e}$, the lowest eigenvalue will be zero, whereas its eigenvector $\vec{X}_3 = \vec{e}$. The eigenvector $\vec{X}_2$, corresponding to the lowest, nonzero eigenvalue, will now be the normal predictor. Typically, constrained

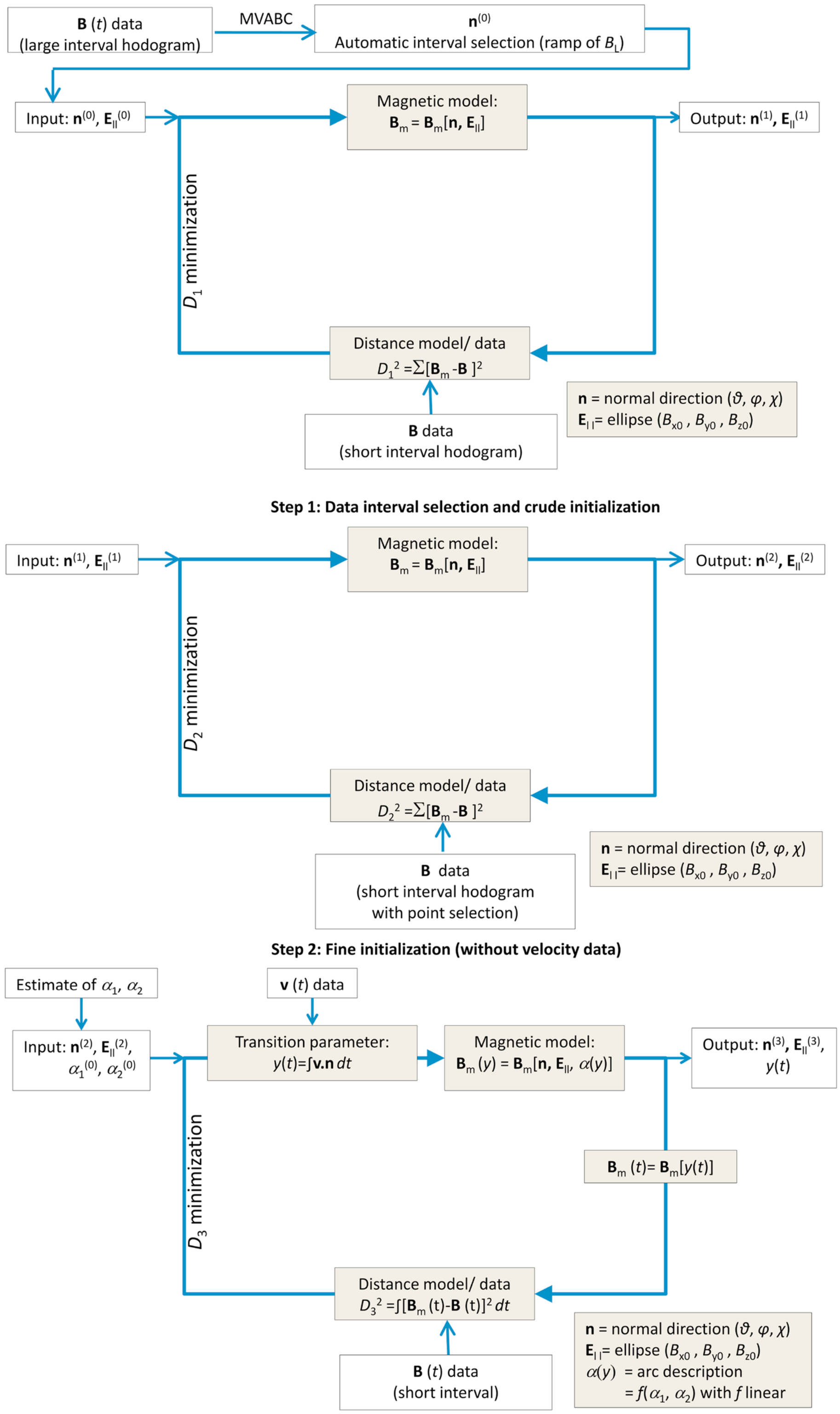

Figure B1. Algorithm used for the initialization of the BV method and the BV method itself. $E_{||}$ stands here for the elliptical form that is determined at each step (characterized by B_{x0}, B_{y0}, and B_{z0}).

variance analysis will give more stable results, even for cases with a small normal component. As seen in section 3.2 it does not perform well for rotational discontinuities, though.

Appendix B: BV Method

The BV method consists of fitting an elliptical model to the magnetic field measured by the satellite. This model (B_{mx}, B_{my}, and B_{mz}) depends explicitly on space, and the way it is described in time therefore depends on the normal velocity.

$$B_{mx} = B_{x0} \cos \alpha \tag{B1}$$

$$B_{my} = B_{y0} \tag{B2}$$

$$B_{mz} = B_{z0} \sin \alpha \tag{B3}$$

with

$$\alpha = \alpha_1 + (\alpha_2 - \alpha_1)\, y/y_{\max}, \tag{B4}$$

In these equations, B_{x0} and B_{z0} are the magnitudes of the major and minor axes of the elliptical model and alpha is the parameter that defines how this ellipse is described. B_{y0} is the normal field. The normal coordinate y is calculated as the integral with respect to time of the measured velocity projected on the normal direction, and $y_{\max}$ is the value of this integral on the whole crossing. The correct frame (and so the normal direction necessary for this projection) is a parameter of the fit, like B_{x0}, B_{y0}, B_{z0}, α_1, and α_2. The distance between the fit and the measured magnetic field is minimized by a numerical algorithm, with respect to all these eight parameters, giving at the end of the process a normal direction, a coordinate along this normal, and an accurate fit of the magnetic field. A careful initialization is necessary. The algorithm used for initialization and the BV method itself is summarized in Figure B1 and explained in full detail in *Dorville et al.* [2014a].

The main assumptions of the method, in addition to the elliptical shape of the field, are the facts that α is supposed to be a linear function of the position along the normal direction and that the shape of y can be obtained by integrating the normal velocity, meaning that the flow across the boundary is assumed negligible with respect to the boundary velocity in the spacecraft frame (if this flow is not negligible, the normal can also be correct at the condition that it is approximately constant). Under these assumptions, the BV method works all the better since the couples of normal components (B_N, and V_N) have more distinguishable profiles as compared to the similar couples on the other components. This is actually a much weaker condition than the one of the usual MVAB method, which implies that the profile of B_N alone can be distinguished from the profiles of the other components of **B**.

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